Announcements

Wednesday, August 23

- Everything you'll need to know is on the master website:

http://people.math.gatech.edu/~cjankowski3/teaching/f2017/m1553/index.html

  or on the website for this section:

  http://people.math.gatech.edu/~jrabinoff/1718F-1553/

  (There is a link from T-Square.) **Read them! Bookmark them!** Chances are, all your (non-math) questions are answered there.
- Warmup assignment is due on Friday at 11:59pm on WeBWorK.
- Enroll in Piazza (the link is on T-Square). You can ask questions there, and we will use it for in-class polling on a daily basis. **Please use your T-Square email address to enroll**, so that your poll responses show up in the T-Square gradebook. **Please join the Piazza group "1553-A and C".**
- It's probably easiest to respond to polls using a smartphone. Download the Piazza app.
- My office is Skiles 244 and my office hours are Monday, 1–3pm and Tuesday, 9–11am.
- Your TAs have office hours too. You can go to any of them. Details on the website.

Chapter 1

Linear Equations

Line, Plane, Space, . . .

Recall that $\mathbf{R}$ denotes the collection of all real numbers, i.e. the number line. It contains numbers like $0, -1, \pi, \frac{3}{2}, \ldots$

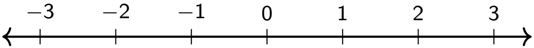
Definition

Let n be a positive whole number. We define

$$\mathbf{R}^n = \text{all ordered } n\text{-tuples of real numbers } (x_1, x_2, x_3, \ldots, x_n).$$

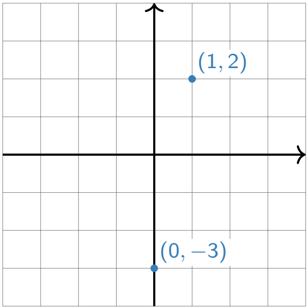
Example

When $n = 1$, we just get $\mathbf{R}$ back: $\mathbf{R}^1 = \mathbf{R}$. Geometrically, this is the *number line*.



Example

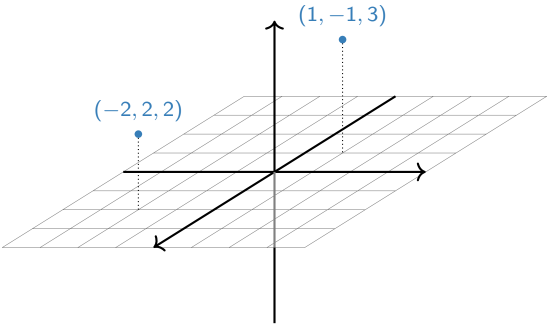
When $n = 2$, we can think of $\mathbf{R}^2$ as the *plane*. This is because every point on the plane can be represented by an ordered pair of real numbers, namely, its x- and y-coordinates.



We can use the elements of $\mathbf{R}^2$ to *label* points on the plane, but $\mathbf{R}^2$ is not defined to be the plane!

Example

When $n = 3$, we can think of $\mathbf{R}^3$ as the *space* we (appear to) live in. This is because every point in space can be represented by an ordered triple of real numbers, namely, its x-, y-, and z-coordinates.

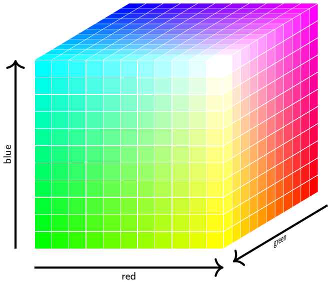


Again, we can use the elements of $\mathbf{R}^3$ to *label* points in space, but $\mathbf{R}^3$ is not defined to be space!

Example

All colors you can see can be described by three quantities: the amount of red, green, and blue light in that color. So we could also think of $\mathbf{R}^3$ as the space of all *colors*:

$$\mathbf{R}^3 = \text{all colors } (r, g, b).$$



Again, we can use the elements of $\mathbf{R}^3$ to *label* the colors, but $\mathbf{R}^3$ is not defined to be the space of all colors!

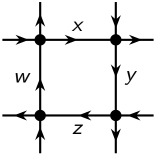
So what is $\mathbf{R}^4$? or $\mathbf{R}^5$? or $\mathbf{R}^n$?

... go back to the *definition*: ordered n-tuples of real numbers

$$(x_1, x_2, x_3, \ldots, x_n).$$

They're still "geometric" spaces, in the sense that our intuition for $\mathbf{R}^2$ and $\mathbf{R}^3$ sometimes extends to $\mathbf{R}^n$, but they're harder to visualize.

Last time we could have used $\mathbf{R}^4$ to label the amount of traffic (x, y, z, w) passing through four streets.



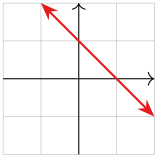
We'll make definitions and state theorems that apply to any $\mathbf{R}^n$, but we'll only draw pictures for $\mathbf{R}^2$ and $\mathbf{R}^3$.

Section 1.1

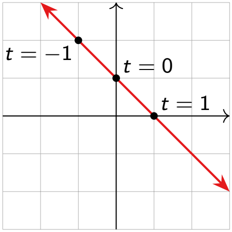
Systems of Linear Equations

One Linear Equation

What does the solution set of a linear equation look like?



$x + y = 1$ ⇝ a line in the plane: $y = 1 - x$
This is called the **implicit equation** of the line.



We can write the same line in **parametric form** in $\mathbf{R}^2$:

$$(x,\ y) = (t,\ 1 - t) \qquad t \text{ in } \mathbf{R}.$$

This means that every point on the line has the form $(t, 1 - t)$ for some real number t.

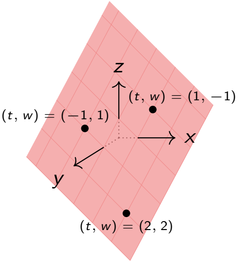
Aside

What is a line? A ray that is *straight* and infinite in both directions.

One Linear Equation
Continued

What does the solution set of a linear equation look like?

$x + y + z = 1 \rightsquigarrow$ a plane in space:
This is the **implicit equation** of the plane.



[interactive]

Does this plane have a **parametric form**?

$$(x, y, z) = (t,\ w,\ 1 - t - w) \qquad t, w \text{ in } \mathbf{R}.$$

Note: we are *labeling* the points on the plane by elements (t, w) in $\mathbf{R}^2$.

Aside
What is a plane? A flat sheet of paper that's infinite in all directions.

One Linear Equation
Continued

What does the solution set of a linear equation look like?

$x + y + z + w = 1 \rightsquigarrow$ a “3-plane” in “4-space”... [not pictured here]

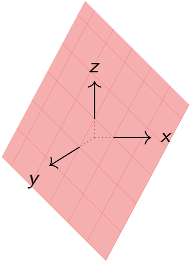
Poll

Everybody get out your gadgets!

Poll

Is the plane from the previous example equal to $\mathbf{R}^2$?

A. Yes **B.** No



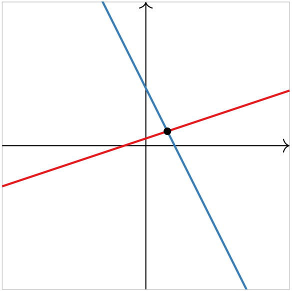
No! Every point on this plane is in $\mathbf{R}^3$: that means it has three coordinates. For instance, $(1, 0, 0)$. Every point in $\mathbf{R}^2$ has two coordinates. But we can *label* the points on the plane by $\mathbf{R}^2$.

Systems of Linear Equations

What does the solution set of a *system* of more than one linear equation look like?

$$x - 3y = -3$$
$$2x + y = 8$$

. . . is the *intersection* of two lines, which is a *point* in this case.



In general it's an intersection of lines, planes, etc.

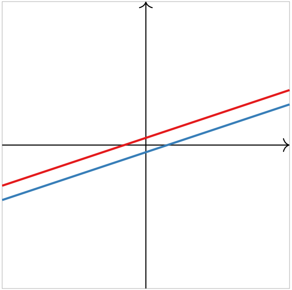
[two planes intersecting]

Kinds of Solution Sets

In what other ways can two lines intersect?

$$x - 3y = -3$$
$$x - 3y = 3$$

has no solution: the lines are *parallel*.



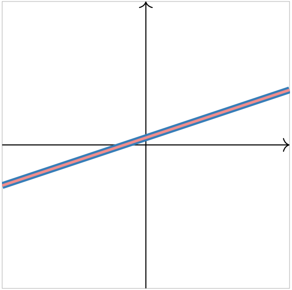
A system of equations with no solutions is called **inconsistent**.

Kinds of Solution Sets

In what other ways can two lines intersect?

$$\begin{aligned} x - 3y &= -3 \\ 2x - 6y &= -6 \end{aligned}$$

has infinitely many solutions: they are the *same line*.



Note that multiplying an equation by a nonzero number gives the *same solution set*. In other words, they are *equivalent* (systems of) equations.

Poll

What about in three variables?

Poll

In how many different ways can three planes intersect in space?

A. One
B. Two
C. Three
D. Four
E. Five
F. Six
G. Seven
H. Eight