Math 1553 Worksheet: Lines and planes in $\mathbf{R}^n$ and §1.1

1. Which of the following equations are linear? Justify your answers.

   a) $3x_1 + \sqrt{x_2} = 4$

   b) $x_1 = x_2 - x_3 + 10x_4$.

   c) $\pi x + \ln(13)y + z = \sqrt[3]{2}$

2. Find all values of h so that the lines $x + hy = -5$ and $2x - 8y = 6$ do *not* intersect.

3. For each of the following, answer true or false. Justify your answer.

   a) Every system of linear equations has at least one solution.

   b) There is a system of linear equations that has exactly 5 solutions.

   c) If a, b, and c are real numbers, then the equation $ax + by = c$ for (x, y, z) in $\mathbf{R}^3$ describes a line.

4. The picture below represents the temperatures at the nodes of a mesh.
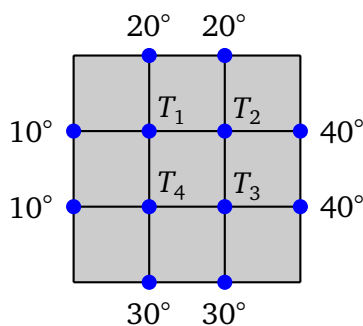
Let $T_1, \ldots, T_4$ be the temperatures at the interior nodes. Suppose that the temperature at each node is the average of the four nearest nodes. For example,

$$T_1 = \frac{10 + 20 + T_2 + T_4}{4}.$$

   a) Write a system of four linear equations whose solution would give the temperatures $T_1, \ldots, T_4$.

   b) Write an augmented matrix that represents that system of equations.

5. Consider the following three planes, where we use (x, y, z) to denote points in $\mathbf{R}^3$:

$$\begin{aligned} 2x + 4y + 4z &= 1 \\ 2x + 5y + 2z &= -1 \\ y + 3z &= 8. \end{aligned}$$

Do all three of the planes intersect? If so, do they intersect at a single point, a line, or a plane?