Math 1553 Quiz: §1.1 Solutions

1. [3 points] Give an example of a system of two linear equations in two variables that has infinitely many solutions.

Solution.

There are many answers. One is

$$\begin{array}{rrr} x + & y = 1 \\ 2x + 2y = 2. \end{array}$$

2. [1 point each] For each matrix, decide if it is in row echelon form (REF), reduced row echelon form (RREF), or neither. Circle the correct answer.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \operatorname{RREF} \begin{pmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \operatorname{RREF} \begin{pmatrix} 1 & 3 & 0 & | & 4 \\ 0 & 0 & 1 & | & -1 \\ 0 & 0 & 0 & | & 0 \end{pmatrix} \operatorname{RREF} \left(\begin{array}{c} 1 & 3 & 0 & | & 4 \\ 0 & 0 & 1 & | & -1 \\ 0 & 0 & 0 & | & 0 \end{array} \right) \operatorname{REF} \operatorname{RREF} \left(\begin{array}{c} 1 & 0 & 0 & | & 1 \\ 0 & 0 & 0 & | & 0 \end{array} \right) \operatorname{RREF} \operatorname{RREF} \left(\begin{array}{c} 1 & 1 & 1 & 1 \\ 1 \\ 1 \\ 1 \end{array} \right) \operatorname{RREF} \operatorname{RREF} \left(\begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array} \right) \operatorname{RREF} \operatorname{RREF} \operatorname{RREF} \right)$$

Solution.

The first matrix is in REF. The second, third, and fourth are in RREF. The last is not in echelon form.

3. [2 points] Give two different ways of making the top-left entry of the following matrix into 1 using a single row operation.

$$\begin{pmatrix} 3 & 0 & 2 \\ 1 & 14 & -7 \\ -2 & -2 & -3 \end{pmatrix}$$

Solution.

One possibility is to replace R_1 by $R_1 + R_3$. Another is to swap the first two rows.