MATH 1553
QUIZ #2: §1.2

Name		**Section**	

1. [5 points] Put the following matrix into reduced row echelon form using elementary row operations. Show your work.

$$\begin{pmatrix} 1 & -1 & 1 & -1 \\ 3 & -4 & 5 & -4 \\ -2 & 1 & 1 & -2 \\ 4 & -2 & -2 & 4 \end{pmatrix}$$

Solution.

$$\begin{pmatrix} 1 & -1 & 1 & -1 \\ 3 & -4 & 5 & -4 \\ -2 & 1 & 1 & -2 \\ 4 & -2 & -2 & 4 \end{pmatrix} \xrightarrow{R_2 = R_2 - 3R_1} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & -1 & 2 & -1 \\ -2 & 1 & 1 & -2 \\ 4 & -2 & -2 & 4 \end{pmatrix}$$

$$\xrightarrow{R_3 = R_3 + 2R_1} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & 3 & -4 \\ 4 & -2 & -2 & 4 \end{pmatrix}$$

$$\xrightarrow{R_4 = R_4 - 4R_1} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & 3 & -4 \\ 0 & 2 & -6 & 8 \end{pmatrix}$$

$$\xrightarrow{R_2 = R_2 \times -1} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -2 & 1 \\ 0 & -1 & 3 & -4 \\ 0 & 2 & -6 & 8 \end{pmatrix}$$

$$\xrightarrow{R_3 = R_3 + R_2} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -3 \\ 0 & 2 & -6 & 8 \end{pmatrix}$$

$$\xrightarrow{R_4 = R_4 - 2R_2} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & -2 & 6 \end{pmatrix}$$

$$\xrightarrow{R_4 = R_4 + 2R_3} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{R_2 = R_2 + 2R_3} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{R_1 = R_1 - R_3} \begin{pmatrix} 1 & -1 & 0 & 2 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{R_1 = R_1 + R_2} \begin{pmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

2. [5 points] True or false: a system of 3 linear equations in 4 variables can have a unique solution. Explain your answer.

Solution.

False. Such a system corresponds to an augmented matrix with 5 columns and 3 rows. If the last column is a pivot column, then the system is inconsistent. Otherwise, one other column is not a pivot column, which means there is a free variable, and hence infinitely many solutions.