

**MATH 1553**  
**QUIZ #2: §1.2**

<b>Name</b>		<b>Section</b>	
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1. [5 points] Put the following matrix into reduced row echelon form using elementary row operations. Show your work.

$$\begin{pmatrix} 1 & -1 & 1 & -1 \\ 3 & -4 & 5 & -4 \\ -2 & 1 & 0 & 1 \\ 4 & -2 & 0 & -2 \end{pmatrix}$$

**Solution.**

$$\begin{pmatrix} 1 & -1 & 1 & -1 \\ 3 & -4 & 5 & -4 \\ -2 & 1 & 0 & 1 \\ 4 & -2 & 0 & -2 \end{pmatrix} \begin{array}{l} R_2 = R_2 - 3R_1 \\ \hline \end{array} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & -1 & 2 & -1 \\ -2 & 1 & 0 & 1 \\ 4 & -2 & 0 & -2 \end{pmatrix}$$

$$\begin{array}{l} R_3 = R_3 + 2R_1 \\ \hline \end{array} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & 2 & -1 \\ 4 & -2 & 0 & -2 \end{pmatrix}$$

$$\begin{array}{l} R_4 = R_4 - 4R_1 \\ \hline \end{array} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & 2 & -1 \\ 0 & 2 & -4 & 2 \end{pmatrix}$$

$$\begin{array}{l} R_2 = R_2 \times -1 \\ \hline \end{array} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -2 & 1 \\ 0 & -1 & 2 & -1 \\ 0 & 2 & -4 & 2 \end{pmatrix}$$

$$\begin{array}{l} R_3 = R_3 + R_2 \\ \hline \end{array} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & -4 & 2 \end{pmatrix}$$

$$\begin{array}{l} R_4 = R_4 - 2R_2 \\ \hline \end{array} \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{array}{l} R_1 = R_1 + R_2 \\ \hline \end{array} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

2. [5 points] True or false: a system of 4 linear equations in 3 variables can have a unique solution. Explain your answer.

**Solution.**

**True.** For instance, the system of equations

$$x = 0$$

$$y = 0$$

$$z = 0$$

$$x + y + z = 0$$

has the unique solution  $(x, y, z) = (0, 0, 0)$ .