MATH 1553
QUIZ #3: §1.3

Name		Section	

1. [1 point each] For each of the following sets of vectors, circle the word that describes the span.

$$\left\{\begin{pmatrix}0\\0\end{pmatrix}\right\}$$ point / line / plane / space

$$\left\{\begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}-3\\-3\end{pmatrix}\right\}$$ point / line / plane / space

$$\left\{\begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}1\\-2\\3\end{pmatrix}\right\}$$ point / line / plane / space

$$\left\{\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix}\right\}$$ point / line / plane / space

$$\left\{\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix}\right\}$$ point / line / plane / space

Solution.

The zero vector spans only a point. The second set consists of collinear vectors, so it spans a line. The third set consists of two noncollinear vectors, which therefore span a plane. The fourth set consists of three coplanar, noncollinear vectors, so it spans a plane. The last set spans all of space.

2. [2 points each] Which of the following vectors are in the span of $\begin{pmatrix}1\\-1\\2\end{pmatrix}$ and $\begin{pmatrix}6\\2\\-1\end{pmatrix}$?

$$\textbf{a)}\ \begin{pmatrix}-4\\-4\\5\end{pmatrix} \qquad \textbf{b)}\ \begin{pmatrix}4\\12\\-5\end{pmatrix} \qquad \textbf{c)}\ \begin{pmatrix}0\\0\\0\end{pmatrix}$$

(Show at least a bit of your work.)

Solution.

a) Deciding if $\begin{pmatrix}-4\\-4\\5\end{pmatrix}$ is a linear combination of $\begin{pmatrix}1\\-1\\2\end{pmatrix}$ and $\begin{pmatrix}6\\2\\-1\end{pmatrix}$ amounts to row reducing the augmented matrix:

$$\left(\begin{array}{cc|c}1 & 6 & -4\\-1 & 2 & -4\\2 & -1 & 5\end{array}\right) \xrightarrow{\text{row reduce}} \left(\begin{array}{cc|c}1 & 6 & -4\\0 & 8 & -8\\0 & 0 & 0\end{array}\right).$$

We can stop here, because we can already see that the system is consistent. (You weren't asked to find the coefficients in a linear combination.)

b) As before, we row reduce

$$\left(\begin{array}{cc|c}1 & 6 & 4\\-1 & 2 & 12\\2 & -1 & -5\end{array}\right) \xrightarrow{\text{row reduce}} \left(\begin{array}{cc|c}1 & 6 & 4\\0 & 1 & 2\\0 & 0 & 13\end{array}\right).$$

We can stop here, because this corresponds to an inconsistent linear system (the last equation being $0 = 13$).

c) The zero vector is in any span:

$$\begin{pmatrix}0\\0\\0\end{pmatrix} = 0\begin{pmatrix}1\\-1\\2\end{pmatrix} + 0\begin{pmatrix}6\\2\\-1\end{pmatrix}.$$