MATH 1553
QUIZ #3: §1.3

Name		**Section**	

1. [1 point each] For each of the following sets of vectors, circle the word that describes the span.

$\left\{\begin{pmatrix}1\\1\end{pmatrix}\right\}$ point line plane space

$\left\{\begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}-3\\3\end{pmatrix}\right\}$ point line plane space

$\left\{\begin{pmatrix}1\\2\\3\end{pmatrix}, \begin{pmatrix}-2\\-4\\-6\end{pmatrix}\right\}$ point line plane space

$\left\{\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\2\end{pmatrix}, \begin{pmatrix}1\\0\\2\end{pmatrix}\right\}$ point line plane space

$\left\{\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\2\end{pmatrix}, \begin{pmatrix}1\\1\\2\end{pmatrix}\right\}$ point line plane space

Solution.

A nonzero vector spans a line. The second set consists of noncollinear vectors, so it spans a plane. The third set consists of two collinear vectors, so it spans a line. The fourth set consists of three coplanar, noncollinear vectors, which therefore span a plane. The last set spans all of space.

[over]

2. [2 points each] Which of the following vectors are in the span of $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 2 \\ -1 \end{pmatrix}$?

$$\textbf{a)}\ \begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix} \qquad \textbf{b)}\ \begin{pmatrix} -3 \\ -5 \\ 7 \end{pmatrix} \qquad \textbf{c)}\ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

(Show at least a bit of your work.)

Solution.

a) Deciding if $\begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix}$ is a linear combination of $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 2 \\ -1 \end{pmatrix}$ amounts to row reducing the augmented matrix:

$$\left(\begin{array}{cc|c} 1 & 6 & 3 \\ -1 & 2 & 5 \\ 2 & -1 & 7 \end{array}\right) \xrightarrow{\text{row reduce}} \left(\begin{array}{cc|c} 1 & 6 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 14 \end{array}\right).$$

We can stop here, because we can already see that the system is inconsistent, as the last row corresponds to the equation $0 = 14$.

b) As before, we row reduce

$$\left(\begin{array}{cc|c} 1 & 6 & -3 \\ -1 & 2 & -5 \\ 2 & -1 & 7 \end{array}\right) \xrightarrow{\text{row reduce}} \left(\begin{array}{cc|c} 1 & 6 & -3 \\ 0 & 8 & -8 \\ 0 & 0 & 0 \end{array}\right).$$

We can stop here, because we can already see that the system is consistent. (You weren't asked to find the coefficients in a linear combination.)

c) The zero vector is in any span:

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = 0\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + 0\begin{pmatrix} 6 \\ 2 \\ -1 \end{pmatrix}.$$