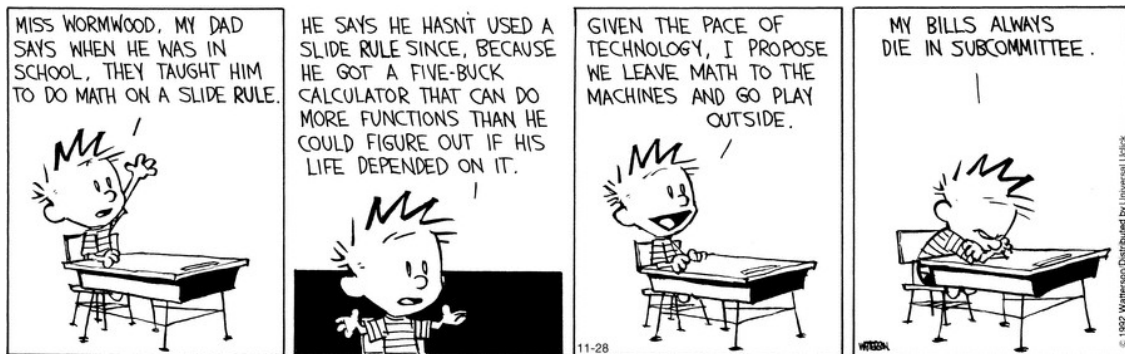


MATH 1553-A MIDTERM EXAMINATION 1

Name		Section	
------	--	---------	--

Please **read all instructions** carefully before beginning.

- Each problem is worth 10 points. The maximum score on this exam is 50 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (notes, text, calculator, etc.) allowed.
- Please show your work.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!



Scoring Page

Please do not write on this page.

1	2	3	4	5	Total

Problem 1.

[2 points each]

In parts (c) and (e), A denotes an $m \times n$ matrix (m rows and n columns), and in (e), b is a vector in \mathbf{R}^m . In (b)–(e), circle **T** if the statement is necessarily true, and circle **F** otherwise.

a) What is the best way to describe the solution set of the equation $x + 2y - z = 0$?

a line in \mathbf{R}^2 a line in \mathbf{R}^3 a plane in \mathbf{R}^2 a plane in \mathbf{R}^3

b) **T** **F** The following matrix has three pivots:

$$\left(\begin{array}{ccc|c} 1 & 7 & 2 & 4 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 15 \end{array} \right)$$

c) **T** **F** It is possible for the matrix equation $Ax = 0$ to be inconsistent.

d) **T** **F** The following matrix corresponds to a linear system with one free variable:

$$\left(\begin{array}{ccc|c} 1 & 7 & 2 & 4 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

e) **T** **F** The solution set of $Ax = b$ is empty or it is a translate of a span in \mathbf{R}^m .

Problem 3.

Consider the following vectors:

$$v_1 = \begin{pmatrix} 17 \\ -3 \\ 24 \end{pmatrix} \quad v_2 = \begin{pmatrix} 7/2 \\ 0 \\ \pi \end{pmatrix}.$$

- a) [4 points] Describe $\text{Span}\{v_1, v_2\}$ geometrically: “it is a in \mathbf{R}^{\square} .”
- b) [6 points] Find a matrix A with three rows, with the property that the matrix equation $Ax = b$ is consistent if and only if b is in $\text{Span}\{v_1, v_2\}$.

Problem 4.

[5 points each]

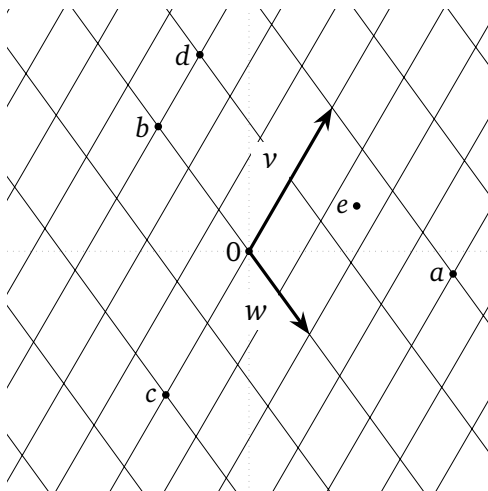
a) Is $\begin{pmatrix} -1 \\ -5 \\ 12 \\ 4 \end{pmatrix}$ in $\text{Span} \left\{ \begin{pmatrix} 1 \\ 3 \\ 4 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 7 \\ 0 \\ 1 \end{pmatrix} \right\}$?

b) Find a vector in \mathbf{R}^3 that is not in $\text{Span} \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$.

Problem 5.

[5 points each]

Consider the following picture of two vectors v, w :



- a) For each of the labeled points, estimate the coefficients x, y such that the linear combination $xv + yw$ is the vector ending at that point.

$$\underline{\hspace{1cm}} v + \underline{\hspace{1cm}} w = a$$

$$\underline{\hspace{1cm}} v + \underline{\hspace{1cm}} w = b$$

$$\underline{\hspace{1cm}} v + \underline{\hspace{1cm}} w = c$$

$$\underline{\hspace{1cm}} v + \underline{\hspace{1cm}} w = d$$

$$\underline{\hspace{1cm}} v + \underline{\hspace{1cm}} w = e$$

- b) Find two vectors p, q in \mathbb{R}^2 such that *none* of the points a, b, c, d, e is in $\text{Span}\{p, q\}$.

You needn't show your work in this problem.

[Scratch work]