# MATH 1553-C
# MIDTERM EXAMINATION 1

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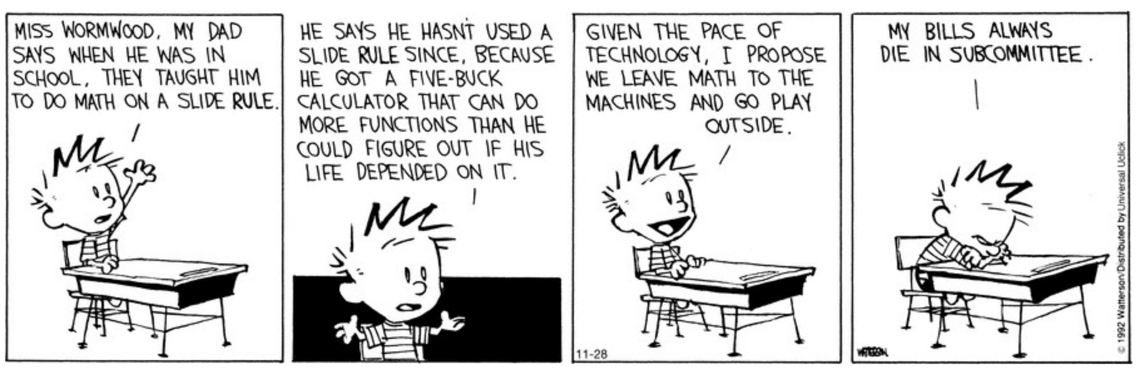
|---|---|---|---|
| **Name** | | **Section** | |

Please **read all instructions** carefully before beginning.

- Each problem is worth 10 points. The maximum score on this exam is 50 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (notes, text, calculator, etc.) allowed.
- Please show your work.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!



## Scoring Page

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| 1 | 2 | 3 | 4 | 5 | Total |
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## Problem 1.

[2 points each]

In parts (c) and (e), $A$ denotes an $m \times n$ matrix ($m$ rows and $n$ columns), and in part (c), $b$ is a vector in $\mathbf{R}^m$. In (b)–(e), circle **T** if the statement is necessarily true, and circle **F** otherwise.

a) What is the best way to describe the solution set of the equation $x + 2y = 0$?

a line in $\mathbf{R}^2$     a line in $\mathbf{R}^3$     a plane in $\mathbf{R}^2$     a plane in $\mathbf{R}^3$

b) **T** **F** The following matrix is in row echelon form:

$$\left(\begin{array}{ccc|c} 1 & 7 & 2 & 4 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 15 \end{array}\right)$$

c) **T** **F** If $A$ has a pivot in every column, then the matrix equation $Ax = b$ is consistent.

d) **T** **F** The following matrix corresponds to a linear system with two free variables:

$$\left(\begin{array}{ccc|c} 1 & 7 & 2 & 4 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

e) **T** **F** The solution set of $Ax = 0$ is a span in $\mathbf{R}^m$.

## Problem 2.

Consider the following system of linear equations:

$$\begin{aligned} 3x + 7y + 4z &= -4 \\ x + 2y + 2z &= -1. \end{aligned}$$

**a)** [1 point ] Write the system as a vector equation.

**b)** [1 point ] Write the system as a matrix equation.

**c)** [1 point ] Write the system as an augmented matrix.

**d)** [4 points] Find the solution set in parametric vector form.

**e)** [3 points] Draw a picture of the solution set.

## Problem 3.

Consider the following vectors:

$$v_1 = \begin{pmatrix} 2\pi \\ -7 \\ 114 \end{pmatrix} \qquad v_2 = \begin{pmatrix} 0 \\ 13 \\ 11/2 \end{pmatrix}.$$

**a)** [4 points] Describe Span$\{v_1, v_2\}$ geometrically: "it is a $\boxed{\phantom{xxxxxxxx}}$ in $\mathbf{R}^{\boxed{\phantom{x}}}$."

**b)** [6 points] Find a matrix $A$ with three rows, with the property that the matrix equation $Ax = b$ is consistent if and only if $b$ is in Span$\{v_1, v_2\}$.

## Problem 4.

[5 points each]

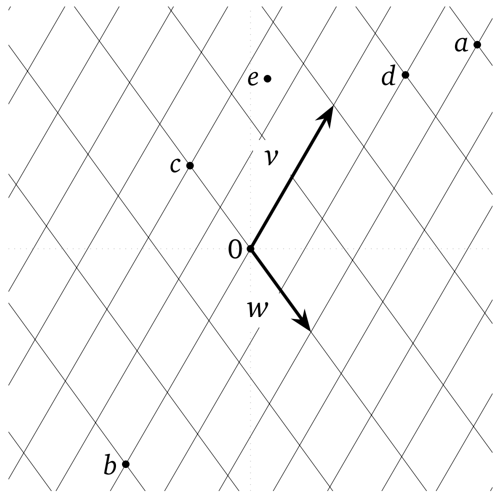
**a)** Is $\begin{pmatrix} 4 \\ 15 \\ -8 \\ -1 \end{pmatrix}$ in Span $\left\{ \begin{pmatrix} 1 \\ 3 \\ 4 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 7 \\ 0 \\ 1 \end{pmatrix} \right\}$?

**b)** Find a vector in $\mathbf{R}^3$ that is not in Span $\left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$.

## Problem 5.

[5 points each]

Consider the following picture of two vectors $v, w$:




**a)** For each of the labeled points, estimate the coefficients $x, y$ such that the linear combination $xv + yw$ is the vector ending at that point.

$$\_\_\_\_\_\_ \, v + \_\_\_\_\_\_ \, w = a$$

$$\_\_\_\_\_\_ \, v + \_\_\_\_\_\_ \, w = b$$

$$\_\_\_\_\_\_ \, v + \_\_\_\_\_\_ \, w = c$$

$$\_\_\_\_\_\_ \, v + \_\_\_\_\_\_ \, w = d$$

$$\_\_\_\_\_\_ \, v + \_\_\_\_\_\_ \, w = e$$

**b)** Find two vectors $p, q$ in $\mathbf{R}^2$ such that *none* of the points $a, b, c, d, e$ is in $\mathrm{Span}\{p, q\}$.

You needn't show your work in this problem.

[Scratch work]