

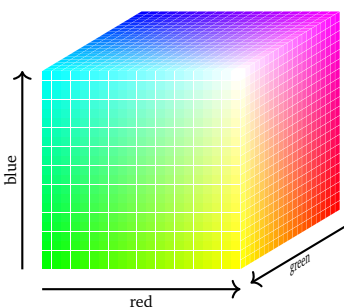
Math 1553 Worksheet §1.7, 1.8, 1.9

1. Justify why each of the following true statements can be checked without row reduction.

a) $\left\{ \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 10 \\ 20 \end{pmatrix}, \begin{pmatrix} 0 \\ 5 \\ 7 \end{pmatrix} \right\}$ is linearly independent.

b) $\left\{ \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 10 \\ 20 \end{pmatrix}, \begin{pmatrix} 0 \\ 5 \\ 7 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$ is linearly dependent.

2. Every color on my computer monitor is a vector in \mathbf{R}^3 with coordinates between 0 and 255, inclusive. The coordinates correspond to the amount of red, green, and blue in the color.



Given colors v_1, v_2, \dots, v_p , we can form a “weighted average” of these colors by making a linear combination

$$v = c_1 v_1 + c_2 v_2 + \dots + c_p v_p$$

with $c_1 + c_2 + \dots + c_p = 1$. Example:

$$\frac{1}{2} \text{ (red square)} + \frac{1}{2} \text{ (blue square)} = \text{ (purple square)}$$

Consider the colors on the right. Are these colors linearly independent? What does this tell you about the colors?

$$\begin{pmatrix} 240 \\ 140 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 120 \\ 100 \end{pmatrix} \quad \begin{pmatrix} 60 \\ 125 \\ 75 \end{pmatrix}$$

3. Let A be a 3×4 matrix with column vectors v_1, v_2, v_3, v_4 . Suppose that $v_2 = 2v_1 - 3v_4$. Find one non-trivial solution to the equation $Ax = 0$.

4. Which of the following transformations T are onto? Which are one-to-one? If the transformation is not onto, find a vector not in the range. If the matrix is not one-to-one, find two vectors with the same image.

a) Counterclockwise rotation by 32° in \mathbf{R}^2 .

b) The transformation $T : \mathbf{R}^3 \rightarrow \mathbf{R}^2$ defined by $T(x, y, z) = (z, x)$.

c) The transformation $T : \mathbf{R}^3 \rightarrow \mathbf{R}^2$ defined by $T(x, y, z) = (0, x)$.

d) The matrix transformation with standard matrix $A = \begin{pmatrix} 1 & 6 \\ -1 & 2 \\ 2 & -1 \end{pmatrix}$.

e) The matrix transformation with standard matrix $A = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$.

5. For each matrix A , describe what the associated matrix transformation T does to \mathbf{R}^3 geometrically.

$$\text{a) } \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{b) } \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

6. The second little pig has decided to build his house out of sticks. His house is shaped like a pyramid with a triangular base that has vertices at the points $(0, 0, 0)$, $(2, 0, 0)$, $(0, 2, 0)$, and $(1, 1, 1)$. The big bad wolf finds the pig's house and blows it down so that the house is rotated by an angle of 45° in a counterclockwise direction about the z -axis, and then projected onto the xy -plane. Find the matrix for this transformation.