MATH 1553-C
QUIZ #4: §§1.7, 1.8, 1.9

Name		**Section**	

1. Consider the vectors

$$v_1 = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \quad v_2 = \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} \quad v_3 = \begin{pmatrix} -4 \\ -2 \\ -8 \end{pmatrix}.$$

Is $\{v_1, v_2, v_3\}$ linearly independent? If not, give an equation of linear dependence.

Solution.

They are linearly dependent: you can see that $v_3 = -2v_1$. Hence an equation of linear dependence is

$$2v_1 + 0v_2 + v_3 = 0.$$

2. Consider the transformation $T\colon \mathbf{R}^3 \to \mathbf{R}^3$ defined by

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -x_1 + x_2 + 2x_3 \\ 2x_1 - 2x_2 + x_3 \\ -x_1 + x_2 + 2x_3 \end{pmatrix}.$$

a) Find the standard matrix A for T.

b) Is T one-to-one? If not, find two vectors in $\mathbf{R}^3$ with the same image.

c) Is T onto? If not, find a vector in $\mathbf{R}^3$ which is not in the range.

Solution.

a) We plug in the unit coordinate vectors:

$$T\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix} \qquad T\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \qquad T\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

$$\implies A = \begin{pmatrix} -1 & 1 & 2 \\ 2 & -2 & 1 \\ -1 & 1 & 2 \end{pmatrix}.$$

b) We need to know if A has a pivot in every column. We row reduce:

$$\begin{pmatrix} -1 & 1 & 2 \\ 2 & -2 & 1 \\ -1 & 1 & 2 \end{pmatrix} \xrightarrow{\text{RREF}} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$

The second column does not have a pivot, so T is not one-to-one. The parametric form of the solution set of $Ax = 0$ is $x_1 = x_2$, $x_3 = 0$. Any value of x_2 gives a solution to $T(x) = 0$, so we have, for instance,

$$T(0) = 0 \qquad T\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 0.$$

c) The first and last entries of $T(x)$ are the same. Therefore, $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ is not in the range, for instance. In particular, T is not onto.