Math 1553 Worksheet §§2.1, 2.2, 2.3

1. If A is a 3×5 matrix and B is a 3×2 matrix, which of the following are defined?

a) $A - B$

b) AB

c) $A^T B$

d) $B^T A$

e) A^2

2. Consider the following linear transformations:

$T : \mathbf{R}^3 \longrightarrow \mathbf{R}^2$ T projects onto the xy-plane, forgetting the z-coordinate

$U : \mathbf{R}^2 \longrightarrow \mathbf{R}^2$ U rotates clockwise by $90°$

$V : \mathbf{R}^2 \longrightarrow \mathbf{R}^2$ V scales the x-direction by a factor of 2.

Let A, B, C be the matrices for T, U, V, respectively.

a) Compute A, B, and C.

b) Compute the matrix for $V \circ U \circ T$.

c) Compute the matrix for $U \circ V \circ T$.

d) Describe U^{-1} and V^{-1}, and compute their matrices.

3. Solve $AB = BC$ for A, assuming A, B, C are $n \times n$ matrices and B is invertible. Be careful!

4. True or false (justify your answer). Answer true if the statement is *always* true. Otherwise, answer false.

a) If A is an $m \times n$ matrix and B is an $n \times p$ matrix, then each column of AB is a linear combination of the columns of A.

b) If A and B are $n \times n$ and both are invertible, then the inverse of AB is $A^{-1}B^{-1}$.

c) If A^T is not invertible, then A is not invertible.

d) If A is an $n \times n$ matrix and the equation $Ax = b$ has at least one solution for each b in $\mathbf{R}^n$, then the solution is *unique* for each b in $\mathbf{R}^n$.

e) If A and B are invertible $n \times n$ matrices, then $A + B$ is invertible and $(A+B)^{-1} = A^{-1} + B^{-1}$.

f) If A and B are $n \times n$ matrices and $ABx = 0$ has a unique solution, then $Ax = 0$ has a unique solution.

5. Consider the matrix

$$A = \begin{pmatrix} 4 & 3 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

a) Compute A^{-1}.

b) Express A^{-1} as a product of elementary matrices.

c) Express A as a product of elementary matrices.