

MATH 1553-A
QUIZ #5: §§2.1–2.3

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1. Let $T: \mathbf{R}^2 \rightarrow \mathbf{R}^2$ be the transformation that rotates counterclockwise by 45° , and let $U: \mathbf{R}^2 \rightarrow \mathbf{R}^2$ be the transformation that scales the x -direction by 2. The matrices A and B for T and U are, respectively:

$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}.$$

- a) Compute the matrices for T^{-1} and U^{-1} .
- b) Compute the matrix for the transformation that first rotates counterclockwise by 45° , then scales the x -direction by 2, then rotates clockwise by 45° .

Solution.

- a) The matrices for T^{-1} and U^{-1} are A^{-1} and B^{-1} , respectively. We compute these using the determinant trick:

$$\det A = \frac{1}{2}(1+1) = 1 \qquad A^{-1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$
$$\det B = 2 \qquad B^{-1} = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}.$$

- b) The transformation in question is $T^{-1} \circ U \circ T$. The matrix for this transformation is

$$A^{-1}BA = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}.$$

2. Let A be an $n \times n$ matrix which is **not invertible**, and let $T(x) = Ax$. Which of the following are definitely true? (Circle all that apply.)
- a) $T(x) = b$ is consistent for all b in \mathbf{R}^n .
 - b) $Ax = 0$ has the trivial solution.
 - c) There exist $x \neq y$ in \mathbf{R}^n such that $T(x) = T(y)$.
 - d) Every vector in \mathbf{R}^n is a linear combination of the columns of A .
 - e) A has at most $n - 1$ pivots.

Solution.

- a) This means that T is onto, which implies A is invertible, so this is **false**.
- b) This is always **true**, whether A is invertible or not.
- c) This means that T is not one-to-one, which is **true** when A is not invertible.
- d) This implies A is invertible, so it is **false**.
- e) This means A does not have n pivots, which is **true** when A is not invertible.