MATH 1553-C
QUIZ #5: §§2.1–2.3

Name		Section	

1. Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the transformation that rotates clockwise by 45°, and let $U : \mathbf{R}^2 \to \mathbf{R}^2$ be the transformation that scales the y-direction by 2. The matrices A and B for T and U are, respectively:

$$A = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}.$$

a) Compute the matrices for T^{-1} and U^{-1}.

b) Compute the matrix for the transformation that first rotates counterclockwise by 45°, then scales the y-direction by 2, then rotates clockwise by 45°.

Solution.

a) The matrices for T^{-1} and U^{-1} are A^{-1} and B^{-1}, respectively. We compute these using the determinant trick:

$$\det A = \frac{1}{2}(1+1) = 1 \qquad A^{-1} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$\det B = 2 \qquad B^{-1} = \frac{1}{2}\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}.$$

b) The transformation in question is $T \circ U \circ T^{-1}$. The matrix for this transformation is

$$ABA^{-1} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}.$$

2. Let A be an $n \times n$ matrix which is **invertible**, and let $T(x) = Ax$. Which of the following are definitely true? (Circle all that apply.)

a) A has at most $n-1$ pivots.

b) There exist $x \neq y$ in $\mathbf{R}^n$ such that $T(x) = T(y)$.

c) $Ax = 0$ has the trivial solution.

d) $T(x) = b$ is consistent for all b in $\mathbf{R}^n$.

e) Every vector in $\mathbf{R}^n$ is a linear combination of the columns of A.

Solution.

a) This means A does not have n pivots, which is **false** when A is invertible.

b) This means that T is not one-to-one, which is **false** when A is invertible.

c) This is always **true**, whether A is invertible or not.

d) This means that T is onto, which is **true** when A is invertible.

e) This is **true** when A is invertible.