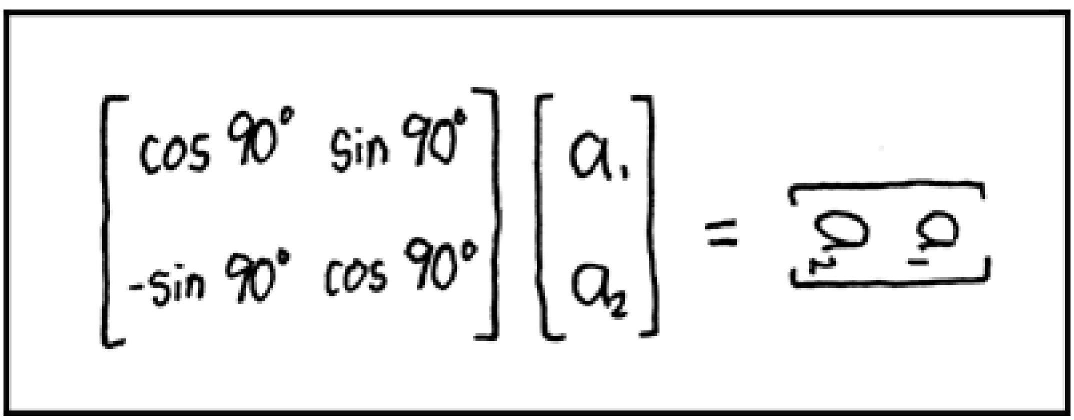
MATH 1553-A
MIDTERM EXAMINATION 2

Name		Section	

Please **read all instructions** carefully before beginning.

- Each problem is worth 10 points. The maximum score on this exam is 50 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (notes, text, calculator, etc.) allowed.
- Please show your work unless indicated otherwise.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!

Hint: this is not worth full credit.



Scoring Page

Please do not write on this page.

1	2	3	4	5	Total

Problem 1.

[2 points each]

In the following problems, A is an $m \times n$ matrix (m rows, n columns).

Circle **T** if the statement is always true, and circle **F** otherwise. You do not need to explain your answer.

a) **T** **F** If A has a pivot in every column, then $\text{Nul} A = \{0\}$.

b) **T** **F** If $m > n$ and $T(x) = Ax$, then T is not one-to-one.

c) **T** **F** A translate of a span is a subspace.

d) **T** **F** There exists a 4×7 matrix A such that $\dim \text{Nul} A = 5$.

e) **T** **F** If $\{v_1, v_2, \ldots, v_n\}$ is a basis for $\mathbf{R}^4$, then $n = 4$.

Problem 2. [2 points each]

Short answer questions: you need not explain your answers.

a) Write a nonzero vector in Col$A$, where $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$.

b) Complete the following definition:

The set $\{v_1, v_2, \ldots, v_m\}$ *is linearly dependent if…*

c) Which of the following are onto transformations? (Check all that apply.)

- ☐ $T : \mathbf{R}^3 \to \mathbf{R}^3$, reflection over the xy-plane
- ☐ $T : \mathbf{R}^3 \to \mathbf{R}^3$, projection onto the xy-plane
- ☐ $T : \mathbf{R}^3 \to \mathbf{R}^2$, project onto the xy-plane, forget the z-coordinate
- ☐ $T : \mathbf{R}^2 \to \mathbf{R}^2$, scale the x-direction by 2

d) Give an example of a subspace of $\mathbf{R}^3$. Be specific.

e) Let A be a square matrix and let $T(x) = Ax$. Which of the following guarantee that T is onto? (Check all that apply.)

- ☐ T is one-to-one
- ☐ $Ax = 0$ is consistent
- ☐ Col$A = \mathbf{R}^n$
- ☐ There is a transformation U such that $T \circ U(x) = x$ for all x

Problem 3.

For each of the following matrices, decide if it is invertible. If it is invertible, find the inverse.

a) [2 points]

$$\begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$$

☐ invertible? Inverse =

b) [2 points]

$$\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$$

☐ invertible? Inverse =

c) [3 points]

$$\begin{pmatrix} 1 & 7 & 3 \\ -2 & -13 & -4 \\ -3 & -23 & -12 \end{pmatrix}$$

☐ invertible? Inverse =

d) [3 points]

$$\begin{pmatrix} 1 & 7 & 3 \\ -2 & -13 & -4 \\ -3 & -23 & -13 \end{pmatrix}$$

☐ invertible? Inverse =

Problem 4.

[5 points each]

Consider $T : \mathbf{R}^2 \to \mathbf{R}^3$ defined by

$$T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+2y \\ 2x+y \\ x-y \end{pmatrix}$$

and $U : \mathbf{R}^3 \to \mathbf{R}^2$ defined by first projecting onto the xy-plane (forgetting the z-coordinate), then rotating counterclockwise by 90°.

a) Compute the standard matrices A and B for T and U, respectively.

$A =$

$B =$

b) Compute the standard matrices for $T \circ U$ and $U \circ T$.

$T \circ U$:

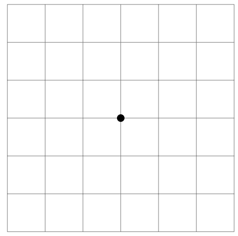
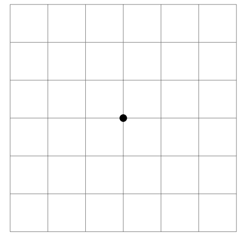
$U \circ T$:

c) Circle all that apply:

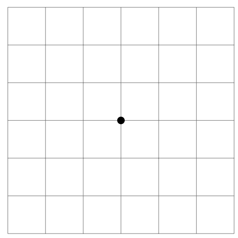
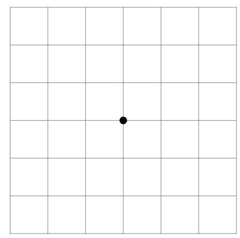
$T \circ U$ is:	one-to-one	onto	invertible
$U \circ T$ is:	one-to-one	onto	invertible

Problem 5. [10 points]

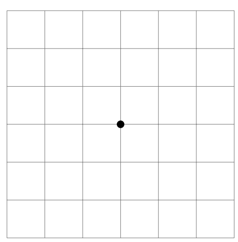
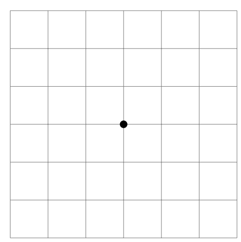
a) Write a 2×2 matrix A with **rank** 2, and draw pictures of $\mathrm{Nul}\,A$ and $\mathrm{Col}\,A$.

$A = \begin{pmatrix} & \\ & \end{pmatrix}$ $\quad \mathrm{Nul}\,A =$  $\quad \mathrm{Col}\,A =$ 

b) Write a 2×2 matrix B with **rank** 1, and draw pictures of $\mathrm{Nul}\,B$ and $\mathrm{Col}\,B$.

$B = \begin{pmatrix} & \\ & \end{pmatrix}$ $\quad \mathrm{Nul}\,B =$  $\quad \mathrm{Col}\,B =$ 

c) Write a 2×2 matrix C with **rank** 0, and draw pictures of $\mathrm{Nul}\,C$ and $\mathrm{Col}\,C$.

$C = \begin{pmatrix} & \\ & \end{pmatrix}$ $\quad \mathrm{Nul}\,C =$  $\quad \mathrm{Col}\,C =$ 

(In the grids, the dot is the origin.)

[Scratch work]