MATH 1553-A

QUIZ #6: §3.1, §3.2

Name		**Section**	

1. [2 points each] Compute the determinants of the following matrices.
[Hint: none require extensive calculations.]

a) $\begin{pmatrix} 1 & 2 & 1 \\ -2 & 0 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

Solution.

For this matrix, it is probably easiest to use the formula for the determinant of a 3×3 matrix:

$$\begin{aligned} \det\begin{pmatrix} 1 & 2 & 1 \\ -2 & 0 & 3 \\ 2 & 3 & 1 \end{pmatrix} &= (1)(0)(3) + (2)(3)(2) + (1)(-2)(3) \\ &\quad - (1)(3)(3) - (2)(-2)(1) - (1)(0)(2) \\ &= 1 \end{aligned}$$

b) $\begin{pmatrix} 0 & 2 & 0 & -4 \\ 2 & 1 & 3 & 0 \\ -2 & 1 & 0 & -1 \\ 0 & 3 & 0 & -2 \end{pmatrix}$

Solution.

Here it is easiest to use cofactor expansion along the third column, then the first column:

$$\det\begin{pmatrix} 0 & 2 & 0 & -4 \\ 2 & 1 & 3 & 0 \\ -2 & 1 & 0 & -1 \\ 0 & 3 & 0 & -2 \end{pmatrix} = -3\det\begin{pmatrix} 0 & 2 & -4 \\ -2 & 1 & -1 \\ 0 & 3 & -2 \end{pmatrix} = -6\det\begin{pmatrix} 2 & -4 \\ 3 & -2 \end{pmatrix} = -48$$

c) $\begin{pmatrix} -1 & 5 & 9 & 12 & 11 \\ 0 & -3 & 11 & 8 & 8 \\ 0 & 0 & 4 & 5 & 1 \\ 0 & 0 & 0 & 3 & 7 \\ 0 & 0 & 0 & 0 & 4 \end{pmatrix}$

Solution.

This is an upper-triangular matrix, so its determinant is the product of the diagonal entries:

$$\det\begin{pmatrix} -1 & 5 & 9 & 12 & 11 \\ 0 & -3 & 11 & 8 & 8 \\ 0 & 0 & 4 & 5 & 1 \\ 0 & 0 & 0 & 3 & 7 \\ 0 & 0 & 0 & 0 & 4 \end{pmatrix} = (-1)(-3)(4)(3)(4) = 144$$

d) $$\begin{pmatrix} 1 & -2 & 28 & -3 & 19 & 92 \\ 0 & 0 & 49 & 57 & -68 & 43 \\ 0 & 0 & 95 & 6 & 74 & 57 \\ -1 & 2 & -68 & 66 & -65 & 2 \\ 0 & 0 & 4 & 45 & 1 & -73 \\ 2 & -4 & 14 & 34 & 1 & 75 \end{pmatrix}$$

Solution.

The second column is a multiple of the first. Hence the columns are linearly dependent, so the matrix is not invertible, so its determinant is zero.

e) $$\begin{pmatrix} d & e & f \\ 2a+3g & 2b+3h & 2c+3i \\ g & h & i \end{pmatrix}^3, \quad \text{assuming } \det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = 1.$$

(Notice the first matrix is cubed.)

Solution.

The matrix $\begin{pmatrix} d & e & f \\ 2a+3g & 2b+3h & 2c+3i \\ g & h & i \end{pmatrix}$ is obtained from $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ by doing one row swap, multiplying one row by 2, and doing one row replacement (not necessarily in that order). Hence

$$\det\begin{pmatrix} d & e & f \\ 2a+3g & 2b+3h & 2c+3i \\ g & h & i \end{pmatrix} = (-1)\cdot 2\cdot \det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = -2.$$

Cubing a matrix cubes its determinant, so

$$\det\begin{pmatrix} d & e & f \\ 2a+3g & 2b+3h & 2c+3i \\ g & h & i \end{pmatrix}^3 = -8.$$