MATH 1553-C
QUIZ #6: §3.1, §3.2

Name		**Section**	

1. [2 points each] Compute the determinants of the following matrices.
[Hint: none require extensive calculations.]

a) $\begin{pmatrix} 2 & 1 & 2 \\ -3 & 0 & 2 \\ 3 & 2 & 1 \end{pmatrix}$

Solution.

For this matrix, it is probably easiest to use the formula for the determinant of a 3×3 matrix:

$$\begin{aligned}\det\begin{pmatrix} 2 & 1 & 2 \\ -3 & 0 & 2 \\ 3 & 2 & 1 \end{pmatrix} &= (2)(0)(1) + (1)(2)(3) + (2)(-3)(2) \\ &\quad - (2)(2)(2) - (1)(-3)(1) - (2)(0)(3) \\ &= -11\end{aligned}$$

b) $\begin{pmatrix} 0 & -4 & 0 & 2 \\ 1 & 3 & 2 & 0 \\ -1 & 2 & 0 & -2 \\ 0 & 2 & 0 & -3 \end{pmatrix}$

Solution.

Here it is easiest to use cofactor expansion along the third column, then the first column:

$$\det\begin{pmatrix} 0 & -4 & 0 & 2 \\ 1 & 3 & 2 & 0 \\ -1 & 2 & 0 & -2 \\ 0 & 2 & 0 & -3 \end{pmatrix} = 2\det\begin{pmatrix} 0 & -4 & 2 \\ -1 & 2 & -2 \\ 0 & 2 & -3 \end{pmatrix} = 2\det\begin{pmatrix} -4 & 2 \\ 2 & -3 \end{pmatrix} = -16$$

c) $\begin{pmatrix} -2 & 9 & 12 & 5 & 11 \\ 0 & -3 & 8 & 11 & 7 \\ 0 & 0 & 2 & 5 & -13 \\ 0 & 0 & 0 & -1 & 8 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}$

Solution.

This is an upper-triangular matrix, so its determinant is the product of the diagonal entries:

$$\det\begin{pmatrix} -2 & 9 & 12 & 5 & 11 \\ 0 & -3 & 8 & 11 & 7 \\ 0 & 0 & 2 & 5 & -13 \\ 0 & 0 & 0 & -1 & 8 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix} = (-2)(-3)(2)(-1)(2) = -24$$

d) $\begin{pmatrix} 1 & 3 & -28 & 92 & 19 & 2 \\ 0 & 57 & 68 & -49 & 43 & 0 \\ 0 & 95 & 74 & 57 & 6 & 0 \\ -1 & -66 & 68 & -65 & 2 & -2 \\ 0 & 45 & 1 & 4 & -73 & 0 \\ 2 & 14 & 1 & 34 & 75 & 4 \end{pmatrix}$

Solution.

The last column is a multiple of the first. Hence the columns are linearly dependent, so the matrix is not invertible, so its determinant is zero.

e) $\begin{pmatrix} d & e & f \\ 2a+3g & 2b+3h & 2c+3i \\ g & h & i \end{pmatrix}^3$, assuming $\det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = 1$.

(Notice the first matrix is cubed.)

Solution.

The matrix $\begin{pmatrix} d & e & f \\ 2a+3g & 2b+3h & 2c+3i \\ g & h & i \end{pmatrix}$ is obtained from $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ by doing one row swap, multiplying one row by 2, and doing one row replacement (not necessarily in that order). Hence

$$\det\begin{pmatrix} d & e & f \\ 2a+3g & 2b+3h & 2c+3i \\ g & h & i \end{pmatrix} = (-1)\cdot 2 \cdot \det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = -2.$$

Cubing a matrix cubes its determinant, so

$$\det\begin{pmatrix} d & e & f \\ 2a+3g & 2b+3h & 2c+3i \\ g & h & i \end{pmatrix}^3 = -8.$$