# MATH 1553-A
QUIZ #7: §5.1, §5.2

Name		Section	

1. [5 points] Write a mathematically correct definition of an eigenvector. Pay attention to your quantifiers.

"v is an eigenvector of an $n \times n$ matrix A provided that

$v \neq 0$ and $Av = \lambda v$ for some scalar λ".

2. [4 points] Consider the matrix A for the transformation that reflects over a line L. Find all eigenvalues of A, and draw a picture of an eigenvector for each eigenvalue in the box below.
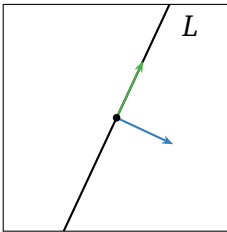
Solution.
The only vectors that are taken to a scalar multiple are the vectors on L, which are not moved, and the vectors perpendicular to L, which are negated. The former have eigenvalue 1, and the latter have eigenvalue -1.

3. [3 points] Find all eigenvalues of A.
$$A = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$$

Solution.
The characteristic polynomial of A is
$$f(\lambda) = \lambda^2 - \text{Tr}(A)\lambda + \det(A) = \lambda^2 - 4\lambda + 1.$$
The roots are
$$\lambda = \frac{4 \pm \sqrt{16-4}}{2} = 2 \pm \sqrt{3}.$$
These are the eigenvalues of A.