MATH 1553-C
QUIZ #7: §5.1, §5.2

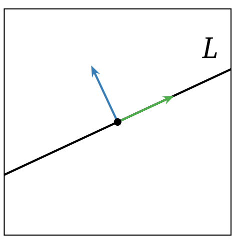
Name		**Section**	

1. [5 points] Write a mathematically correct definition of an eigenvalue. Pay attention to your quantifiers.

"λ is an eigenvalue of an $n \times n$ matrix A provided that

there exists a nonzero solution v to the equation $Av = \lambda v$.".

2. [4 points] Consider the matrix A for the transformation that reflects over a line L. Find all eigenvalues of A, and draw a picture of an eigenvector for each eigenvalue in the box below.



Solution.

The only vectors that are taken to a scalar multiple are the vectors on L, which are not moved, and the vectors perpendicular to L, which are negated. The former have eigenvalue 1, and the latter have eigenvalue -1.

3. [3 points] Find all eigenvalues of A.

$$A = \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}$$

Solution.

The characteristic polynomial of A is

$$f(\lambda) = \lambda^2 - \mathrm{Tr}(A)\lambda + \det(A) = \lambda^2 - 3\lambda - 1.$$

The roots are

$$\lambda = \frac{3 \pm \sqrt{9+4}}{2} = \frac{3 \pm \sqrt{13}}{2}.$$

These are the eigenvalues of A.