MATH 1553-A
MIDTERM EXAMINATION 3

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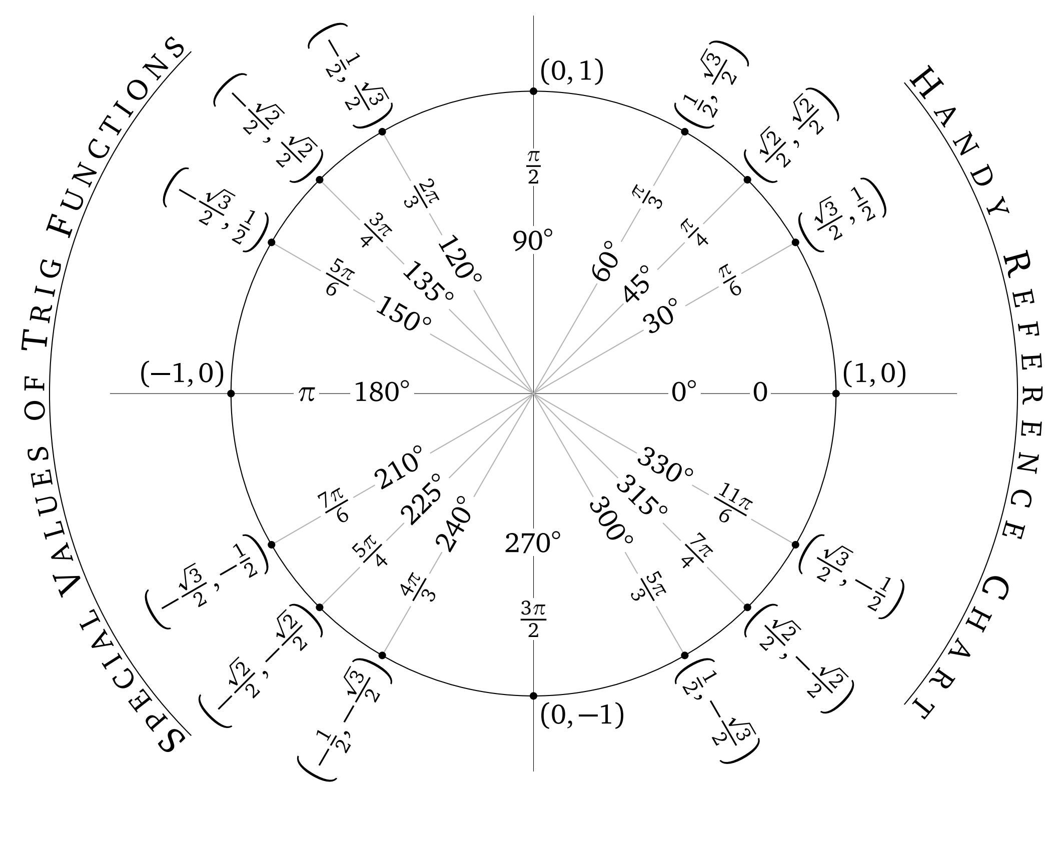
Please **read all instructions** carefully before beginning.

- Please leave your GT ID card on your desk until your TA scans your exam.
- All graded work for Problem *n* must appear on the page containing Problem *n* or the page labeled "Scratch page for Problem *n*".
- Each problem is worth 10 points. The maximum score on this exam is 50 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (notes, text, etc.) allowed.
- Please show your work.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!



[Scratch page for Problem 1]

Problem 1.

[2 points each]

In this problem, if the statement is always true, circle **T**; otherwise, circle **F**.

All matrices are assumed to have real entries.

a) **T** **F** An upper-triangular matrix can have a complex (non-real) eigenvalue.

b) **T** **F** If an $n \times n$ matrix A has a zero eigenvalue, then $\operatorname{rank}(A) < n$.

c) **T** **F** Every upper-triangular matrix is diagonalizable.

d) **T** **F** If A is an $n \times n$ matrix and c is a scalar, then $\det(cA) = c\det(A)$.

e) **T** **F** $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ is similar to $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$.

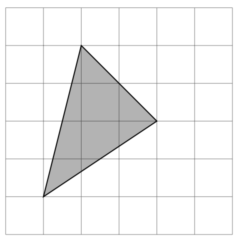
[Scratch page for Problem 2]

Problem 2.

[2 points each]

Short answer problems: you need not explain your work.

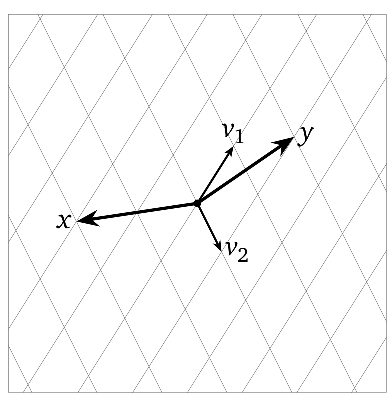
a) What is the area of the triangle in the picture?



b) Give an example of a 2×2 matrix that is neither invertible nor diagonalizable.

c) Give an example of two 2×2 matrices that have the same eigenvalues but are not similar.

d) Suppose that $A = P\begin{pmatrix} 1/2 & 0 \\ 0 & -1 \end{pmatrix}P^{-1}$, where P has columns v_1 and v_2. Given x and y in the picture below, draw the vectors Ax and Ay.



e) With respect to the picture in (d), find the $\mathcal{B}$-coordinates of an eigenvector of A with eigenvalue $1/2$, where $\mathcal{B} = \{v_1, v_2\}$.

[Scratch page for Problem 3]

Problem 3.

[2 points each]

Consider the matrix

$$A = \begin{pmatrix} -2\sqrt{3}-1 & 5 \\ -1 & -2\sqrt{3}+1 \end{pmatrix}$$

a) Find both complex eigenvalues of A.

b) Find an eigenvector corresponding to each eigenvalue.

c) Find an invertible matrix P and a rotation-scale matrix C such that $A = PCP^{-1}$.

d) By what angle does C rotate?

e) Successive multiplication by A:

spirals in　　rotates around an ellipse　　spirals out

(circle the best option).

[Scratch page for Problem 4]

Problem 4.

[10 points]

For which value(s) of a is $\lambda = 1$ an eigenvector of this matrix?

$$A = \begin{pmatrix} 3 & -1 & 0 & a \\ a & 2 & 0 & 4 \\ 2 & 0 & 1 & -2 \\ 13 & a & -2 & -7 \end{pmatrix}$$

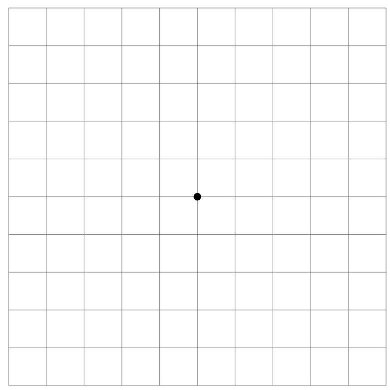
[Scratch page for Problem 5]

Problem 5.

[5 points each]

Let $A = \begin{pmatrix} 2 & -3 \\ 0 & 1 \end{pmatrix}$.

a) Draw all eigenspaces of A, and label them with the corresponding eigenvalue:



b) Compute A^n, where $n \geq 1$ is any whole number. Your answer should be a single 2×2 matrix whose entries are formulas involving n.

[Ungraded scratch page]