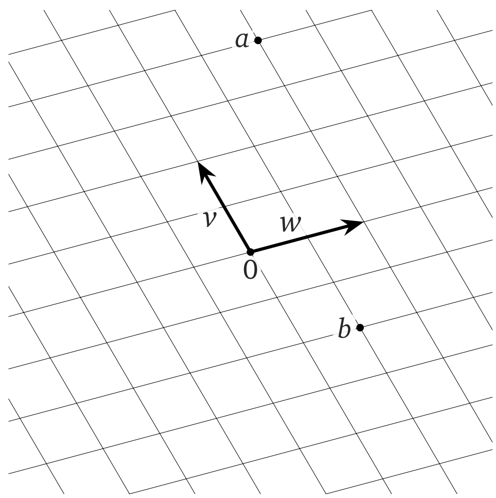
MATH 1553
QUIZ #3: §§3.1, 3.2

Name		Section	

1. [1 point each] Consider the following picture of two vectors v, w:



For each of the labeled points, estimate the coefficients x, y such that the linear combination $xv + yw$ is the vector ending at that point. (Use the parallelogram law for vector addition; you needn't show your work.)

$$\_\_\_\_\_\_ v + \_\_\_\_\_\_ w = a$$

$$\_\_\_\_\_\_ v + \_\_\_\_\_\_ w = b$$

Solution.

As you can tell from the grid, you reach a by following v twice then w once. Hence $a = 2v + w$. Similarly, $b = -v + \frac{1}{2}w$.

2. [3 points] Consider the following vectors:

$$u = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} \qquad v = \begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix} \qquad w = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}.$$

Decide if w is in $\mathrm{Span}\{u, v\}$ by solving a system of equations.

Solution.

To say that w is in $\text{Span}\{u, v\}$ means that w is a linear combination of u and v. Hence We have to solve the vector equation

$$x\begin{pmatrix}1\\0\\3\end{pmatrix} + y\begin{pmatrix}0\\-1\\2\end{pmatrix} = \begin{pmatrix}2\\1\\4\end{pmatrix}.$$

We form an augmented matrix and row reduce:

$$\left(\begin{array}{cc|c}1 & 0 & 2\\0 & -1 & 1\\3 & 2 & 4\end{array}\right) \xrightarrow{\text{REF}} \left(\begin{array}{cc|c}1 & 0 & 2\\0 & -1 & 1\\0 & 0 & 0\end{array}\right).$$

This corresponds to a consistent system of equations, so w is in $\text{Span}\{u, v\}$.

3. [1 point each] For each of the following sets of vectors, circle the word that describes the span.

$\left\{\begin{pmatrix}1\\1\end{pmatrix}\right\}$ point / line / plane / space

$\left\{\begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}0\\0\end{pmatrix}\right\}$ point / line / plane / space

$\left\{\begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\0\\1\end{pmatrix}\right\}$ point / line / plane / space

$\left\{\begin{pmatrix}2\\0\\0\end{pmatrix}, \begin{pmatrix}2\\3\\0\end{pmatrix}, \begin{pmatrix}4\\5\\6\end{pmatrix}\right\}$ point / line / plane / space

$\left\{\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\2\end{pmatrix}, \begin{pmatrix}1\\0\\-2\end{pmatrix}\right\}$ point / line / plane / space

Solution.

A single nonzero vector spans a line. Adding the zero vector to a set does not change the span, so the second set spans the same line. The two nonzero, noncollinear vectors in the third set span a plane. The vectors in the fourth set are the columns of a matrix in row echelon form, with a pivot in each row, so they span space. The three vectors in the fifth set span the xz-plane.