# MATH 1553
# QUIZ #4: §§3.7, 3.9, 4.1

| Name | | Section | |
|---|---|---|---|

**1.** Consider the matrix

$$A = \begin{pmatrix} -6 & -18 & 1 & 8 \\ -5 & -15 & -2 & 1 \\ -1 & -3 & -1 & -1 \end{pmatrix}$$

and the matrix transformation $T(x) = Ax$.

**a)** [1 point ] What is the domain of $T$?

**b)** [1 point ] What is the codomain of $T$?

**c)** [3 points] Find a basis for the range of $T$.

**d)** [2 points] What is the nullity of $A$?

## Solution.

**a)** $\mathbf{R}^4$

**b)** $\mathbf{R}^3$

**c)** The range of $T$ is $\mathrm{Col}A$. To compute a basis for this, we reduce to row echelon form:

$$\begin{pmatrix} -6 & -18 & 1 & 8 \\ -5 & -15 & -2 & 1 \\ -1 & -3 & -1 & -1 \end{pmatrix} \xrightarrow{\text{REF}} \begin{pmatrix} 1 & 3 & 1 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

The first and third columns have pivots, so a basis for $\mathrm{Col}A$ is

$$\left\{ \begin{pmatrix} -6 \\ -5 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} \right\}.$$

**d)** We just computed $\mathrm{rank}A = 2$. By the Rank Theorem,

$$\#\text{columns} = 4 = \mathrm{rank}A + \mathrm{nullity}A = 2 + \mathrm{nullity}A,$$

so $\mathrm{nullity}A = 2$.

**2.** [3 points] Consider the matrix transformation $T : \mathbf{R}^2 \to \mathbf{R}^2$ defined by

$$T(x) = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} x.$$

Draw the image of the **F** under this transformation.