Name		Section	

1. Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation that slides a point diagonally up or down at a 45° angle until it hits the line $y = x$, as in the following picture:
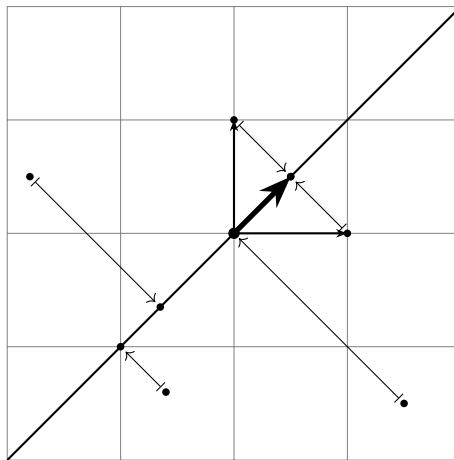
a) [4 points] Compute the standard matrix for T.

b) [3 points] Is T one-to-one? If so, explain why; if not, find two different vectors with the same image.

c) [3 points] Is T onto? If so, explain why; if not, find a vector not in the range.

Solution.

a) Using the diagram, we see that

$$T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = T\begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

These are the columns of the standard matrix

$$\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}.$$

b) No, T is not one-to-one. For instance, $T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = T\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

c) No, T is not onto: its range is the line $y = x$. For instance, $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are not in the range.