Math 1553 Worksheet §4.4, Matrix Operations

1. If A is a 3×5 matrix and B is a 3×2 matrix, which of the following are defined? Very briefly justify your answer.

a) $A - B$

b) AB

c) $A^T B$

d) $B^T A$

e) A^2

2. True or false (justify your answer). Answer true if the statement is *always* true. Otherwise, answer false.

a) If A is an $m \times n$ matrix and B is an $n \times p$ matrix, then each column of AB is a linear combination of the columns of A.

b) If A is a 3×4 matrix and B is a 4×2 matrix, then the linear transformation transformation Z defined by $Z(x) = ABx$ has domain $\mathbf{R}^2$ and codomain $\mathbf{R}^3$.

c) Suppose $T : \mathbf{R}^n \to \mathbf{R}^m$ and $U : \mathbf{R}^m \to \mathbf{R}^p$ are linear transformations and $U \circ T$ is onto. Then U and T must both be onto.

3. Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be rotation *clockwise* by $60°$. Let $U : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation with standard matrix $\begin{pmatrix} -2 & 1 \\ 1 & 0 \end{pmatrix}$.

a) Find the standard matrix K for the composition $U \circ T$.

b) Find the standard matrix L for the composition $T \circ U$.

c) Is rotating clockwise by $60°$ and then performing U, the same as first performing U and then rotating clockwise by $60°$?