MATH 1553
QUIZ #6: §§5.1–5.3

Name		**Section**	

1. [2 points each] In this problem we consider these matrices:

$$A=\begin{pmatrix}2&5&-3\\-2&-5&0\\-5&-4&4\end{pmatrix}\qquad B=\begin{pmatrix}2&5&0&-3\\-2&2&1&-1\\-2&-5&0&0\\-3&-6&-1&5\end{pmatrix}\qquad C=\begin{pmatrix}4&0&0&0\\-5&-1&0&0\\5&-5&-2&0\\1&3&1&-1\end{pmatrix}\qquad D=\begin{pmatrix}-3&-2&3&-1\\1&-4&-1&-3\\-4&-5&4&3\\-1&4&1&-5\end{pmatrix}$$

Compute the following determinants (none require extensive calculations):

a) $\det(A) =$

b) $\det(B) =$

c) $\det(C) =$

d) $\det(D) =$

e) $\det(BC^2B^{-1}) =$

Solution.

a) Expanding cofactors along the third column gives

$$\det(A) = -3\det\begin{pmatrix}-2&-5\\-5&-4\end{pmatrix} + 4\det\begin{pmatrix}2&5\\-2&-5\end{pmatrix} = -3(8-25)+4(0) = 51.$$

b) First we perform a row replacement, which does not change the determinant:

$$\begin{pmatrix}2&5&0&-3\\-2&2&1&-1\\-2&-5&0&0\\-3&-6&-1&5\end{pmatrix} \xrightarrow{R_4=R_4+R_2} \begin{pmatrix}2&5&0&-3\\-2&2&1&-1\\-2&-5&0&0\\-5&-4&0&4\end{pmatrix}$$

Expanding cofactors along the third column of this matrix gives

$$\det(B) = -\det\begin{pmatrix}2&5&-3\\-2&-5&0\\-5&-4&4\end{pmatrix} = -\det(A) = -51.$$

c) This is a triangular matrix, so its determinant is the product of the diagonal entries:

$$\det(C) = (4)(-1)(-2)(-1) = -8.$$

d) The third column of D is a multiple of the first, so D is not invertible, and hence $\det(D) = 0$.

e) We use multiplicativity of the determinant and the fact that $\det(B^{-1}) = \det(B)^{-1}$:

$$\det(BC^2B^{-1}) = \det(B)\det(C)^2\det(B)^{-1} = \det(C)^2 = 64.$$