MATH 1553
QUIZ #7: §§6.1,6.2

Name		**Section**	

1. [3 points] Write a correct definition of an eigenvalue:

"λ is an eigenvalue of an $n \times n$ matrix A provided that

...there exists a nonzero vector v such that $Av = \lambda v$.

2. [4 points] Find all eigenvalues of A, and produce a basis for each eigenspace.

$$A = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$$

Solution.

First we compute the characteristic polynomial:

$$f(\lambda) = \lambda^2 - \mathrm{Tr}(A)\lambda + \det(A) = \lambda^2 - 2\lambda + 1 = (\lambda - 1)^2.$$

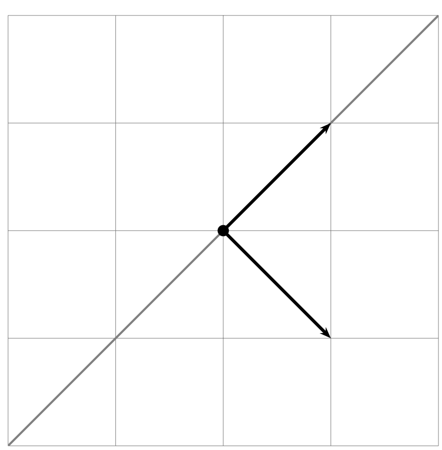
Hence the only eigenvalue is $\lambda = 1$. The eigenspace is the null space of

$$A - 1I = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \xrightarrow{RREF} \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}.$$

There is one free variable, so the 1-eigenspace is spanned by any eigenvector; a basis is

$$\left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}.$$

3. [3 points] Let A be the matrix for the transformation from $\mathbf{R}^2$ to $\mathbf{R}^2$ that reflects over the line $y = x$. Draw an eigenvector of A on the graph below.



Solution.

Any nonzero vector on the line or perpendicular to it is an eigenvector.