

Math 1553 Supplement, §3.1 and §3.2

1. Consider the augmented matrix

$$\left(\begin{array}{ccc|c} 2 & -2 & 2 & 0 \\ 1 & -3 & -4 & -9 \\ 3 & -1 & 8 & 9 \end{array} \right)$$

Question: Does the corresponding linear system have a solution? If so, what is the solution set?

- a) Formulate this question as a vector equation.
- b) Formulate this question as a system of linear equations.
- c) What does this mean in terms of spans?
- d) Answer the question using the [interactive demo](#).
- e) Answer the question using row reduction.
- f) Find a **different** solution in parts (e) and (d).

2. Let $v_1 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ $v_2 = \begin{pmatrix} -2 \\ -3 \\ -1 \end{pmatrix}$ $w = \begin{pmatrix} 2 \\ -4 \\ 8 \end{pmatrix}$.

Question: Is w a linear combination of v_1 and v_2 ? In other words, is w in $\text{Span}\{v_1, v_2\}$?

- a) Formulate this question as a vector equation.
- b) Formulate this question as a system of linear equations.
- c) Formulate this question as an augmented matrix.
- d) Answer the question using the [interactive demo](#).
- e) Answer the question using row reduction.

3. Let

$$A = \begin{pmatrix} 1 & 0 & 5 \\ -2 & 1 & -6 \\ 0 & 2 & 8 \end{pmatrix}, \quad b = \begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix}$$

Is b in the span of the columns of A ? In other words, is b a linear combination of the columns of A ? Justify your answer.

4. Consider the vector equation

$$x \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + y \begin{pmatrix} -2 \\ -1 \\ -1 \end{pmatrix} + z \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} -5 \\ -1 \\ -2 \end{pmatrix}.$$

Question: Is there a solution? If so, what is the solution set?

- a) Formulate this question as an augmented matrix.

- b) Formulate this question as a system of linear equations.
 - c) What does this mean in terms of spans?
 - d) Answer the question using the [interactive demo](#).
 - e) Answer the question using row reduction.
5. Decide if each of the following statements is true or false. If it is true, prove it; if it is false, provide a counterexample.
- a) Every set of four or more vectors in $\mathbf{R}^3$ will span $\mathbf{R}^3$.
 - b) The span of any set contains the zero vector.