MATH 1553, FALL 2018
SAMPLE MIDTERM 1: THROUGH SECTION 3.4

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Please **read all instructions** carefully before beginning.

- Please leave your GT ID card on your desk until your TA scans your exam.

- Each problem is worth 10 points. The maximum score on this exam is 50 points.

- You have 50 minutes to complete this exam.

- There are no aids of any kind (notes, text, calculator, etc.) allowed.

- Please show your work.

- You may cite any theorem proved in class or in the sections we covered in the text.

- Good luck!

> This is a practice exam. It is meant to be similar in format, length, and difficulty to the real exam. It is **not** meant as a comprehensive list of study problems. I recommend completing the practice exam in 50 minutes, without notes or distractions.

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Problem 1.

[2 points each]

a) Compute: $\begin{pmatrix} 3 & 2 \\ -2 & 0 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ -3 \end{pmatrix} =$

The remaining problems are True or false. Circle **T** if the statement is **always** true, and circle **F** otherwise. You do not need to justify your answer.

b) **T** **F** The matrix $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ is in reduced row echelon form.

c) **T** **F** If the augmented matrix corresponding to a linear system of equations has a pivot in every row, then the system is consistent.

d) **T** **F** If A is an $m \times n$ matrix and $Ax = 0$ has a unique solution, then $Ax = b$ is consistent for every b in $\mathbf{R}^m$.

e) **T** **F** The three vectors $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, and $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ span $\mathbf{R}^3$.

[Scratch work for problem 1]

Problem 2.

a) [2 points] If A is a 2×3 matrix with 2 pivots, then the set of solutions to $Ax = 0$ is a:
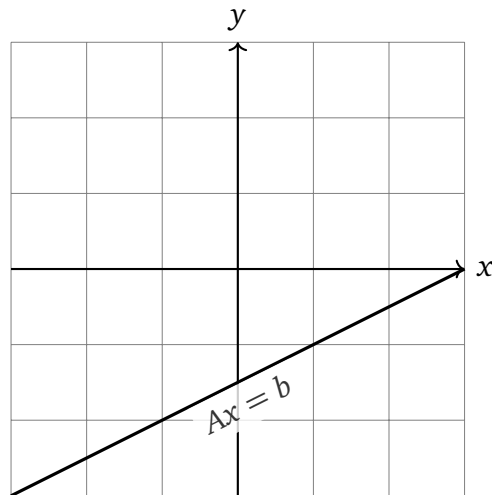
(circle one answer)   **point**   **line**   **plane**   **3-plane**

in:

(circle one answer)   **R** $\mathbf{R}^2$ $\mathbf{R}^3$.

b) [2 points] Write a vector equation which represents an inconsistent system of two linear equations in x_1 and x_2.

c) [3 points] For some 2×2 matrix A and vector b in $\mathbf{R}^2$, the solution set of $Ax = b$ is drawn below. Draw the solution set of $Ax = 0$.

d) [3 points] If b, v, w are vectors in $\mathbf{R}^3$ and $\mathrm{Span}\{b, v, w\} = \mathbf{R}^3$, is it possible that b is in $\mathrm{Span}\{v, w\}$? Justify your answer.

[Scratch work for problem 2]

Problem 3.

Johnny Rico believes that the secret to the universe can be found in the system of two linear equations in x and y given by

$$x - \ \ y = h$$
$$3x + hy = 4$$

where h is a real number.

a) [5 points] Find all values of h (if any) which make the system inconsistent. Briefly justify your answer.

b) [5 points] Find all values of h (if any) which make the system have a unique solution. Briefly justify your answer.

[Scratch work for problem 3]

Problem 4.

a) [6 points] Find the parametric form of the general solution of the following system of equations. Clearly indicate which variables (if any) are free variables.

$$\begin{aligned}
x_1 + 2x_2 + 2x_3 - x_4 &= 4 \\
2x_1 + 4x_2 + x_3 - 2x_4 &= -1 \\
-x_1 - 2x_2 - x_3 + x_4 &= -1
\end{aligned}$$

b) [4 points] Write the solution set of

$$\begin{aligned}
x_1 + 2x_2 + 2x_3 - x_4 &= 0 \\
2x_1 + 4x_2 + x_3 - 2x_4 &= 0 \\
-x_1 - 2x_2 - x_3 + x_4 &= 0
\end{aligned}$$

in parametric vector form.

[Scratch work for problem 4]

Problem 5.

The diagram below represents traffic in a city.

Traffic flow (cars/hr)
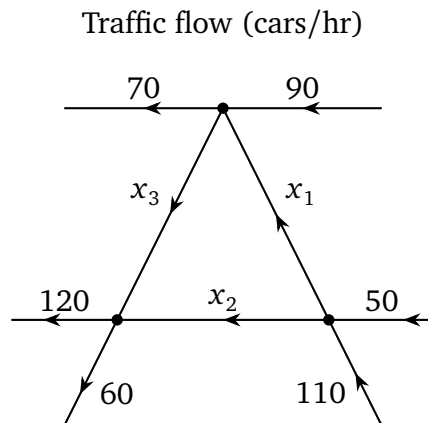
a) [5 points] Write a system of three linear equations whose solution would give the values of x_1, x_2, and x_3. Do not solve it.

b) [5 points] Write the system of equations as a vector equation. Do not solve it.

[Scratch work for problem 5]

[Additional scratch work]

[Additional scratch work]