MATH 1553-C
MIDTERM EXAMINATION 1

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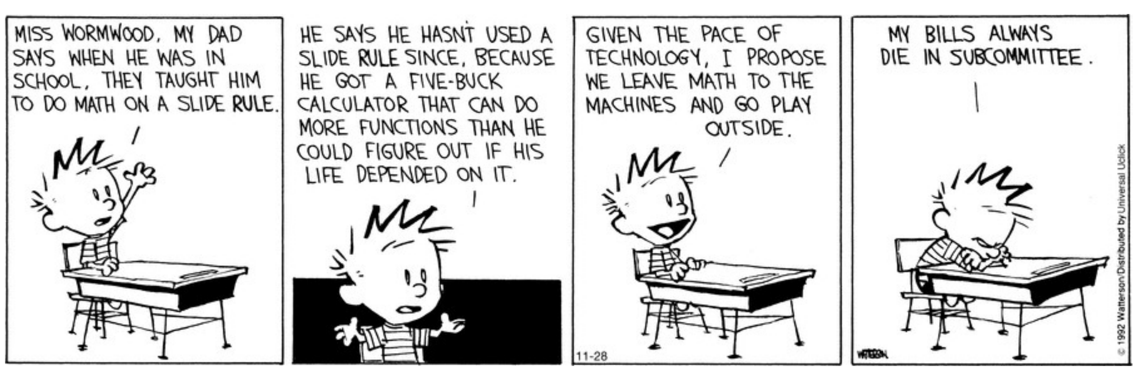
Please **read all instructions** carefully before beginning.

- Please leave your GT ID card on your desk until your TA scans your exam.
- Each problem is worth 10 points. The maximum score on this exam is 50 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (notes, text, calculator, etc.) allowed.
- Please show your work.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!



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Problem 1.

[2 points each]

Parts (a) and (b) refer to the following matrix:

$$A = \begin{pmatrix} 1 & -2 \\ 2 & -4 \\ -1 & 2 \end{pmatrix}.$$

a) What is the best way to describe the span of the columns A?

a line in $\mathbf{R}^2$ a line in $\mathbf{R}^3$ a plane in $\mathbf{R}^2$ a plane in $\mathbf{R}^3$

b) What is the best way to describe the solution set of $Ax = 0$?

a line in $\mathbf{R}^2$ a line in $\mathbf{R}^3$ a plane in $\mathbf{R}^2$ a plane in $\mathbf{R}^3$

In the following questions, circle **T** if the statement is necessarily true, and circle **F** otherwise.

c) **T**   **F**   The following matrix is in row echelon form:

$$\left(\begin{array}{ccc|c} 1 & 7 & 2 & 4 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 15 \end{array}\right)$$

d) **T**   **F**   If A is a 2×3 matrix, then $Ax = b$ can have a unique solution.

e) **T**   **F**   If A is an $m \times n$ matrix, then the solution set of $Ax = b$ is empty or it is a span in $\mathbf{R}^n$.

[Scratch work for problem 1]

Problem 2. [2 points each]

In this problem, it is not necessary to show your work or justify your answers.

a) Compute the product (your answer will be a single vector):

$$\begin{pmatrix} 1 & 3 & -1 \\ 2 & 0 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} =$$

b) Give an example of a 3×4 matrix A such that $Ax = b$ is consistent for all b in $\mathbf{R}^3$.

c) Find a matrix A with three rows such that $Ax = b$ is consistent if and only if b is a linear combination of the vectors

$$\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix}.$$

d) Find three vectors u, v, w in $\mathbf{R}^3$ whose span is

$$\{(x, y, 0) \mid x, y \text{ are in } \mathbf{R}\} \qquad \text{(the } xy\text{-plane).}$$

e) If A is a 2×2 matrix such that $A\binom{1}{0} = \binom{-1}{2}$ and $A\binom{0}{1} = \binom{3}{3}$, then what is $A\binom{1}{-2}$?

[Scratch work for problem 2]

Problem 3.

Consider the following system of linear equations:

$$\begin{aligned} -2x_1 + x_2 - 4x_3 + x_4 &= 4 \\ -x_1 + x_2 - x_3 \phantom{{}+x_4} &= 1 \\ x_2 + 2x_3 - x_4 &= -2 \end{aligned}$$

a) [2 points] Write the system as a vector equation.

b) [2 points] Write the system as a matrix equation.

c) [1 point] Write the system as an augmented matrix.

d) [2 points] Row-reduce this matrix to reduced row echelon form.

e) [2 points] Write the parametric vector form of the general solution.

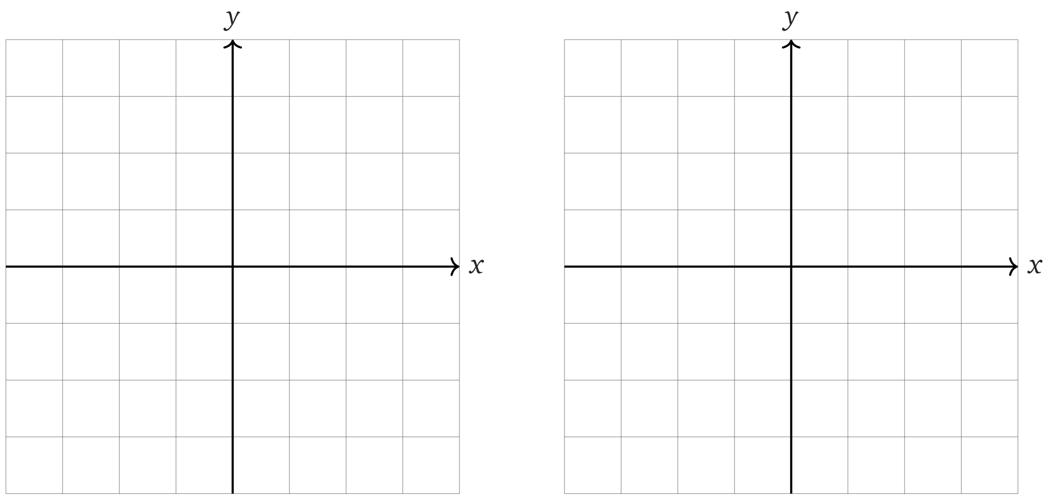
f) [1 point] Write any solution of the original equation.

[Scratch work for problem 3]

Problem 4.

In this problem, we consider the matrix

$$A = \begin{pmatrix} 1 & 2 \\ -2 & -4 \end{pmatrix}.$$



On the grid on the **left**, draw and label:

a) [3 points] The solution set of $Ax = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$. If the system is inconsistent, write “inconsistent”.

b) [2 points] The solution set of $Ax = 0$. If the system is inconsistent, write “inconsistent”.

On the grid on the **right**, draw and label:

c) [3 points] The span of the columns of A.

d) [2 points] A vector b such that $Ax = b$ is inconsistent. If no such b exists, explain why.

[Scratch work for problem 4]

Problem 5.

a) [6 points] Find all values of k such that

$$\begin{pmatrix}1\\k\\6\end{pmatrix} \quad \text{is in} \quad \text{Span}\left\{\begin{pmatrix}2\\0\\4\end{pmatrix}, \begin{pmatrix}-1\\3\\2\end{pmatrix}\right\}.$$

b) [4 points] For every number k that you found in (a), express $\begin{pmatrix}1\\k\\6\end{pmatrix}$ as a linear combination of $\begin{pmatrix}2\\0\\4\end{pmatrix}$ and $\begin{pmatrix}-1\\3\\2\end{pmatrix}$.

[Scratch work for problem 5]

[Additional scratch work]

[Additional scratch work]