

Math 1553, Extra Practice for Midterm 2 (sections 3.5-4.4)

1. Circle **T** if the statement is always true, and circle **F** otherwise. You do not need to explain your answer.

- a) **T** **F** If $\{v_1, v_2, v_3, v_4\}$ is a basis for a subspace V of $\mathbf{R}^n$, then $\{v_1, v_2, v_3\}$ is a linearly independent set.
- b) **T** **F** If A is an $n \times n$ matrix and $Ae_1 = Ae_2$, then the homogeneous equation $Ax = 0$ has infinitely many solutions.
- c) **T** **F** The solution set of a consistent matrix equation $Ax = b$ is a subspace.
- d) **T** **F** There exists a 3×5 matrix with rank 4.
- e) **T** **F** If A is an 9×4 matrix with a pivot in each column, then
 $\text{Nul}A = \{0\}$.
- f) **T** **F** If A is a matrix with more rows than columns, then the transformation $T(x) = Ax$ is not one-to-one.
- g) **T** **F** A translate of a span is a subspace.
- h) **T** **F** There exists a 4×7 matrix A such that $\text{nullity}A = 5$.
- i) **T** **F** If $\{v_1, v_2, \dots, v_n\}$ is a basis for $\mathbf{R}^4$, then $n = 4$.

2. Short answer questions: you need not explain your answers.

a) Write a nonzero vector in $\text{Col}A$, where $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$.

b) Complete the following definition:

A transformation $T : \mathbf{R}^n \rightarrow \mathbf{R}^m$ is one-to-one if...

c) Which of the following are onto transformations? (Check all that apply.)

☐ $T : \mathbf{R}^3 \rightarrow \mathbf{R}^3$, reflection over the xy -plane

☐ $T : \mathbf{R}^3 \rightarrow \mathbf{R}^3$, projection onto the xy -plane

☐ $T : \mathbf{R}^3 \rightarrow \mathbf{R}^2$, project onto the xy -plane, forget the z -coordinate

☐ $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$, scale the x -direction by 2

d) Let A be a square matrix and let $T(x) = Ax$. Which of the following guarantee that T is onto? (Check all that apply.)

☐ T is one-to-one

☐ $Ax = 0$ is consistent

☐ $\text{Col}A = \mathbf{R}^n$

☐ There is a transformation U such that $T \circ U(x) = x$ for all x

3. Parts (a) and (b) are unrelated.

a) Consider $T : \mathbf{R}^3 \rightarrow \mathbf{R}^3$ given by

$$T(x, y, z) = (x, x + z, 3x - 4y + z, x).$$

Is T one-to-one? Justify your answer.

b) Find all real numbers h so that the transformation $T : \mathbf{R}^3 \rightarrow \mathbf{R}^2$ given by

$$T(v) = \begin{pmatrix} -1 & 0 & 2-h \\ h & 0 & 3 \end{pmatrix} v$$

is onto.

4. a) Determine which of the following transformations are linear.
- (1) $S : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ given by $S(x_1, x_2) = (x_1, 3 + x_2)$
 - (2) $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ given by $T(x_1, x_2) = (x_1 - x_2, x_1 x_2)$
 - (3) $U : \mathbf{R}^2 \rightarrow \mathbf{R}^3$ given by $U(x_1, x_2) = (-x_2, x_1, 0)$
- b) Complete the following definition (be mathematically precise!):
A set of vectors $\{v_1, v_2, \dots, v_p\}$ in $\mathbf{R}^n$ is *linearly independent* if...
- c) If $\{v_1, v_2, v_3\}$ are vectors in $\mathbf{R}^3$ with the property that none of the vectors is a scalar multiple of another, is $\{v_1, v_2, v_3\}$ necessarily linearly independent? Justify your answer.
5. Let $T : \mathbf{R}^3 \rightarrow \mathbf{R}^2$ be the linear transformation which projects onto the yz -plane and then forgets the x -coordinate, and let $U : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ be the linear transformation of rotation counterclockwise by 60° . Their standard matrices are

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad B = \frac{1}{2} \begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix},$$

respectively.

- a) Which composition makes sense? (Circle one.)
- $$U \circ T \qquad T \circ U$$
- b) Find the standard matrix for the transformation that you circled in (b).
6. Consider the following matrix A and its reduced row echelon form:
- $$\begin{pmatrix} 2 & 4 & 7 & -16 \\ 3 & 6 & -1 & -1 \\ 5 & 10 & 6 & -17 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$
- a) Find a basis for $\text{Col}A$.
- b) Find a basis $\mathcal{B}$ for $\text{Nul}A$.
- c) For each of the following vectors v , decide if v is in $\text{Nul}A$, and if so, write x as a linear combination of your basis from part (b).

$$\begin{pmatrix} 7 \\ 3 \\ 1 \\ 2 \end{pmatrix} \qquad \begin{pmatrix} -5 \\ 2 \\ -2 \\ -1 \end{pmatrix}$$

7. Consider $T : \mathbf{R}^2 \rightarrow \mathbf{R}^3$ defined by

$$T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + 2y \\ 2x + y \\ x - y \end{pmatrix}$$

and $U: \mathbf{R}^3 \rightarrow \mathbf{R}^2$ defined by first projecting onto the xy -plane (forgetting the z -coordinate), then rotating counterclockwise by 90° .

a) Compute the standard matrices A and B for T and U , respectively.

b) Compute the standard matrices for $T \circ U$ and $U \circ T$.

c) Circle all that apply:

$T \circ U$ is: one-to-one onto

$U \circ T$ is: one-to-one onto

8. a) Write a 2×2 matrix A with **rank** 2, and draw pictures of $\text{Nul } A$ and $\text{Col } A$.

$$A = \begin{pmatrix} & \\ & \end{pmatrix} \quad \text{Nul } A = \begin{array}{|c|c|c|c|c|c|c|} \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline \end{array} \quad \text{Col } A = \begin{array}{|c|c|c|c|c|c|c|} \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline \end{array}$$

b) Write a 2×2 matrix B with **rank** 1, and draw pictures of $\text{Nul } B$ and $\text{Col } B$.

$$B = \begin{pmatrix} & \\ & \end{pmatrix} \quad \text{Nul } B = \begin{array}{|c|c|c|c|c|c|c|} \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline \end{array} \quad \text{Col } B = \begin{array}{|c|c|c|c|c|c|c|} \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline \end{array}$$

c) Write a 2×2 matrix C with **rank** 0, and draw pictures of $\text{Nul } C$ and $\text{Col } C$.

$$C = \begin{pmatrix} & \\ & \end{pmatrix} \quad \text{Nul } C = \begin{array}{|c|c|c|c|c|c|c|} \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline \end{array} \quad \text{Col } C = \begin{array}{|c|c|c|c|c|c|c|} \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline & & & & & & \\ \hline \end{array}$$

(In the grids, the dot is the origin.)