MATH 1553-C
MIDTERM EXAMINATION 2

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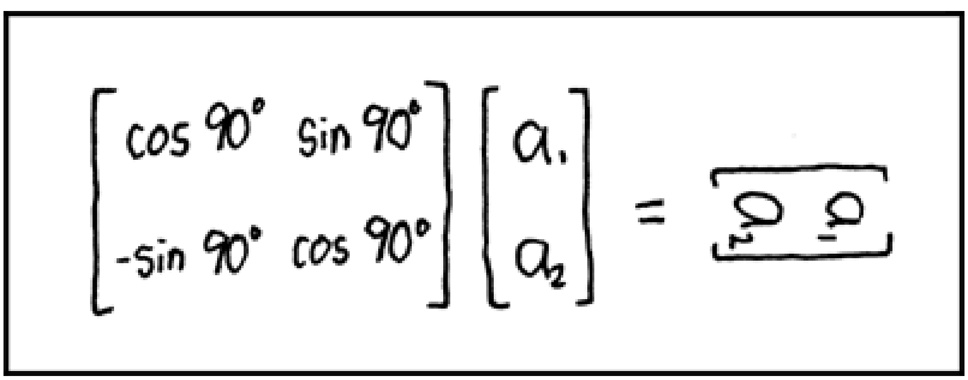
Please **read all instructions** carefully before beginning.

- Please leave your GT ID card on your desk until your TA scans your exam.
- Each problem is worth 10 points. The maximum score on this exam is 50 points.
- You have 50 minutes to complete this exam.
- There are no aids of any kind (notes, text, calculator, etc.) allowed.
- Please show your work unless instructed otherwise.
- You may cite any theorem proved in class or in the sections we covered in the text.
- Good luck!



(This is a joke, not a hint.)

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Problem 1.

[2 points each]

a) Which of the following are subspaces of $\mathbf{R}^3$? Circle all that apply.

(i) the solution set of $\begin{cases} x - 2y = 0 \\ 2x - 4y = 0 \\ -x + 2y = 0 \end{cases}$ (ii) $\mathrm{Col}\begin{pmatrix} 1 & -2 \\ 2 & -4 \\ -1 & 2 \end{pmatrix}$ (iii) $\mathrm{Nul}\begin{pmatrix} 1 & 2 & 3 \\ -2 & -4 & -6 \end{pmatrix}$

(iv) $\mathrm{Span}\left\{\begin{pmatrix} 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 2 \\ -4 \end{pmatrix}, \begin{pmatrix} 3 \\ -6 \end{pmatrix}\right\}$ (v) the solution set of $\begin{pmatrix} 1 & 2 & 3 \\ -2 & -4 & -6 \end{pmatrix} x = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

(vi) the range of a linear transformation with codomain $\mathbf{R}^3$

In the following questions, circle **T** if the statement is necessarily true, and circle **F** otherwise.

b) **T** **F** There exists a 3×4 matrix A such that $\mathrm{Col}A$ is a plane and $\mathrm{Nul}A$ is a line.

c) **T** **F** If $T : \mathbf{R}^3 \to \mathbf{R}^4$ is a one-to-one linear transformation, then the set of vectors $\{T(e_1), T(e_2), T(e_3)\}$ is a basis for the range of T.

d) **T** **F** The span of four vectors has dimension 4.

e) **T** **F** If A is an $n \times n$ matrix and $Ae_1 = Ae_2$, then $\mathrm{Col}A$ is not equal to $\mathbf{R}^n$.

[Scratch work for problem 1]

Problem 2.

[2 points each]

In this problem, it is not necessary to show your work or justify your answers.

a) Give an example of a linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^2$ that is not onto.

b) Give a basis of the xy-plane in $\mathbf{R}^3$ (the subspace consisting of all vectors with third coordinate equal to zero).

c) Write a nonzero vector in Nul$A$, where $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \\ -1 & -2 \end{pmatrix}$.

d) Complete the following definition:

The dimension of a subspace V of $\mathbf{R}^n$ is ...

e) Which of the following are one-to-one transformations? (Check all that apply.)

- ☐ $T : \mathbf{R}^3 \to \mathbf{R}^3$, clockwise rotation by 32° around the z-axis
- ☐ $T : \mathbf{R}^3 \to \mathbf{R}^3$, projection onto the xy-plane (i.e. set the z-coordinate to 0)
- ☐ $T : \mathbf{R}^3 \to \mathbf{R}^2$, $T(x, y, z) = (x + y, x - z)$
- ☐ $T : \mathbf{R}^2 \to \mathbf{R}^3$, $T(x, y) = (x, y, y)$

[Scratch work for problem 2]

Problem 3.

Consider the following vectors:

$$v_1 = \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix} \quad v_2 = \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} \quad v_3 = \begin{pmatrix} 2 \\ 5 \\ -5 \end{pmatrix}$$

and the subspace $V = \text{Span}\{v_1, v_2, v_3\}$.

a) [4 points] Is $\{v_1, v_2, v_3\}$ linearly independent? If so, explain why; if not, produce an equation of linear dependence.

b) [3 points] Find a basis for V.

c) [2 points] Find another basis for V that does not include a multiple of any of the vectors in your answer for (b).

d) [1 points] What is $\dim V$?

[Scratch work for problem 3]

Problem 4.

In this problem we consider two linear transformations:

$T\colon \mathbf{R}^2 \to \mathbf{R}^2$ is defined as follows: first it flips the plane over the x-axis, then rotates the plane counterclockwise by 45°, then stretches the x-direction by a factor of 2.

$U\colon \mathbf{R}^2 \to \mathbf{R}^3$ is defined by the formula $U\begin{pmatrix} x \\ y \end{pmatrix} = x\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} + y\begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}$.

a) [3 points] Compute the standard matrix for T.

b) [2 points] Compute the standard matrix for U.

c) [2 points] Compute the standard matrix for $U \circ T$.

d) [3 points] Which of the following transformations are onto? (Circle all that apply.)

T $\qquad$ U $\qquad$ $U \circ T$

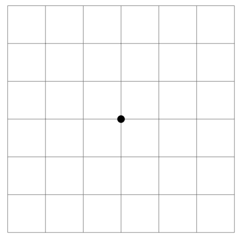
[Scratch work for problem 4]

Problem 5. [10 points]

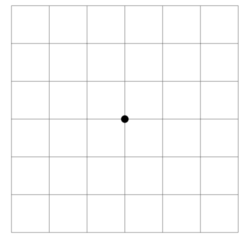
a) Write a 2×2 matrix A such that **the range of** $T(x) = Ax$ **is a point**, and draw pictures of $\mathrm{Nul}A$ and $\mathrm{Col}A$.

$A = \begin{pmatrix} & \\ & \end{pmatrix}$

Nul A =



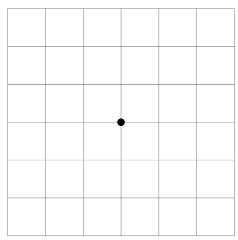
Col A =



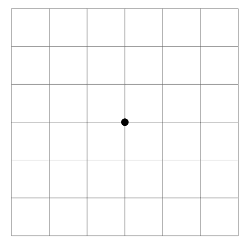
b) Write a 2×2 matrix B such that $\mathrm{Nul}\,B$ **is a line**, and draw pictures of $\mathrm{Nul}\,B$ and $\mathrm{Col}\,B$.

$B = \begin{pmatrix} & \\ & \end{pmatrix}$

Nul B =



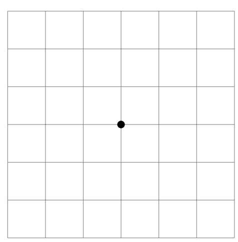
Col B =



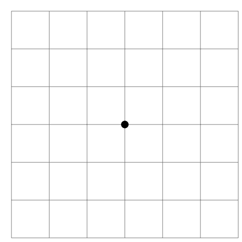
c) Write a 2×2 matrix C with **linearly independent columns**, and draw pictures of $\mathrm{Nul}\,C$ and $\mathrm{Col}\,C$.

$C = \begin{pmatrix} & \\ & \end{pmatrix}$

Nul C =



Col C =



(In the grids, the dot is the origin.)

[Scratch work for problem 5]

[Additional scratch work]

[Additional scratch work]