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1. (a) $(2+i)+(2-i)=4$; $(2+i)(2-i)=5$.
   (b) $(-1+i)+(-1+i)=-2+2i$; $(-1+i)(-1+i)=-2+I$
   (c) $(\cos\theta+i\sin\theta)+(\cos\theta-i\sin\theta)=2\cos\theta$; $(\cos\theta+i\sin\theta)(\cos\theta-i\sin\theta)=1$

5. 30°: $\frac{\sqrt{3}}{2}+\frac{1}{2}i$; 60°: $\frac{1}{2}+\frac{\sqrt{3}}{2}i$; 90°: i; 120°: $-\frac{1}{2}+\frac{\sqrt{3}}{2}i$. $w=\frac{\sqrt{3}}{2}+\frac{1}{2}i$, so $w^2=\frac{3}{4}-\frac{1}{2}+\frac{\sqrt{3}}{4}i+\frac{\sqrt{3}}{4}i=\frac{1}{2}+\frac{\sqrt{3}}{2}i$. $w^{12}=1$.

6. If $z=r\cos\theta+ir\sin\theta$, then $\frac{1}{z}$ has absolute value $\underline{\frac{1}{r}}$ and angle $\underline{-\theta}$. Its polar form is $\underline{\frac{1}{r}e^{-i\theta}}$.

7. $\begin{pmatrix}1 & -3\\ 3 & 1\end{pmatrix}\begin{pmatrix}1\\ -3\end{pmatrix}=\begin{pmatrix}10\\ 0\end{pmatrix}$. $(1+3i)(1-3i)=1^2+3^2=10+0i$.

8. $\begin{pmatrix}A_1 & -A_2\\ A_2 & A_1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}=\begin{pmatrix}b_1\\ b_2\end{pmatrix}$. These are block matrices and vectors.

9. (a) $2+i$
   (b) $(2+i)(1-i)$
   (c) $e^{-\frac{i\pi}{2}}=-i$
   (d) $e^{-i\pi}=-1$
   (e) $\frac{1-i}{1+i}=\frac{(1-i)(1-i)}{(1+i)(1-i)}=\frac{-2i}{2}=-i$
   (f) $(-i)^{103}=i$

10. The sum $z+\bar{z}$ is always real. The difference $z-\bar{z}$ is always imaginary. Assume $z\neq 0$. The product $z\times\bar{z}$ is always real (the length or absolute value squared). The ratio $\frac{z}{\bar{z}}$ has absolute value 1.

12. (a) If $a=b=d=1$, the eigenvalues are complex when c is negative.
    (b) When $ad=bc$, the eigenvalues are 0 and $a+d$.

18. Cube roots of 1 are 1, $e^{\frac{2\pi}{3}}$, and $e^{\frac{4\pi}{3}}$. Cube roots of -1 are -1, $e^{\frac{\pi}{3}}$, and $e^{\frac{5\pi}{3}}$. Together these form the sixth roots of 1. Note that these are all point on the unit circle in the complex plane, with angles of 0°, 120°, and 240° (for roots of 1), and 180°, 60°, and 300° (roots of -1).

19. $(\cos\theta+i\sin\theta)^3=\cos^3\theta+3i\cos^2\theta\sin\theta-3\cos\theta\sin^2\theta-i\sin^3\theta$. But since

$$(\cos\theta+i\sin\theta)^3=(e^{i\theta})^3=e^{3i\theta}=\cos 3\theta+i\sin 3\theta,$$

we get

$$\begin{aligned}\cos 3\theta &= \cos^3\theta-3\cos\theta\sin^2\theta\\ &= \cos^3\theta-3\cos\theta(1-\cos^2\theta)\\ &= 4\cos^3\theta-3\cos\theta\end{aligned}$$

, and

$$\begin{aligned}\sin 3\theta &= 3\cos^2\theta\sin\theta - \sin^3\theta \\ &= 3(1-\sin^2\theta)\sin\theta - \sin^3\theta \\ &= 3\sin\theta - 4\sin^3\theta\end{aligned}$$

20. $\frac{1}{z} = \frac{\bar{z}}{|z|^2}$. So if $\frac{1}{z} = \bar{z}$, then $|z| = 1$, which means z is on the unit circle.