Homework #6

due Thursday, October 10

Exercises from Strang:

Problem Set 3.3 #24, 25, 34
Problem Set 3.4 #2, 7, 8, 12, 21, 22, 45[1]
Problem Set 3.5 #3, 4, 11, 18, 19, 24, 25
Problem Set 4.1 #9, 10(a), 14, 17, 21

Additional Problem:

1. Use the Sage Cell Server at

https://sagecell.sagemath.org/

to find the RREF of the following 7×10 matrix. Also find the RREF of its transpose.

$$A = \begin{pmatrix} -2 & -2 & 1 & 2 & -4 & -4 & 1 & -8 & 9 & 1 \\ -4 & -3 & 0 & 0 & 11 & -5 & -9 & 5 & -5 & 3 \\ -5 & -5 & 1 & 3 & -1 & -8 & -3 & -10 & 8 & 6 \\ 3 & 3 & -1 & -2 & 2 & 5 & 0 & 9 & -9 & -4 \\ 4 & 4 & 0 & 1 & -11 & 4 & 8 & -4 & 7 & -10 \\ 2 & 2 & 0 & -3 & 5 & 4 & 1 & 6 & -3 & 1 \\ 3 & 3 & 0 & -3 & 3 & 6 & 2 & 8 & -5 & -1 \end{pmatrix}$$

You can copy and paste the following code to initialize the matrix:

```
A=matrix(
   [[-2,-2, 1, 2, -4,-4, 1, -8, 9,  1],
    [-4,-3, 0, 0, 11,-5,-9,  5,-5,  3],
    [-5,-5, 1, 3, -1,-8,-3,-10, 8,  6],
    [ 3, 3,-1,-2,  2, 5, 0,  9,-9, -4],
    [ 4, 4, 0, 1,-11, 4, 8, -4, 7,-10],
    [ 2, 2, 0,-3,  5, 4, 1,  6,-3,  1],
    [ 3, 3, 0,-3,  3, 6, 2,  8,-5, -1]])
```

To find the RREF and transpose of a matrix A:

```
A.rref()
A.T
```

Then answer the following questions:

[1]You may **not** use the dimension formula from Question 43, as we haven't proved it. Instead, write down a basis for V and a basis for W, then figure out why combining the two bases gives a linearly dependent set. Lastly, use the formal definition of linear dependence and a bit of algebra to find a non-zero vector that must be in both spaces. You may also want to do #4.1.14 first.

a) What is rank(A)?

b) Find bases for each of the four fundamental spaces of A.

c) The following code creates a 7×11 matrix of zeros, then inserts A into its first 10 columns and the vector $\vec{c} = (-6, -12, -18, 13, 22, -1, 6)^T$ into its last column, creating the augmented matrix $\left(A \mid \vec{c}\,\right)$:

```
B = matrix(7,11)
B[:,:10]=A
B[:,10]=vector([-6, -12, -18, 13, 22, -1, 6])
```

Use the RREF of the augmented matrix to find a particular solution $\vec{x}_p$ of $A\vec{x} = \vec{c}$.

d) Since $\vec{x}_p \in \mathbf{R}^{10}$, it can be written as linear combination of basis vectors of which two spaces?

e) Explain how you would go about expressing $\vec{x}_p$ as a linear combination of those basis vectors. (For extra props, or for some masochistic fun: carry this out... please don't do it by hand... the numbers are very nasty...)