Homework #7

due Tuesday, October 15

Exercises from Strang:

Problem Set 4.1 #24, 28, 29
Problem Set 4.2 #2, 5–7, 11, 12, 17[1], 19, 20, 24, 26, 31
Problem Set 4.3 #28

Additional Problem:

1. You may use Sage for any or all parts of this question. Be sure to show all your work. For example, if you used Sage to compute the RREF of a matrix, show the RREF and explain what conclusions you draw from it that answer the relevant question.

https://sagecell.sagemath.org/

Consider the equation $A\vec{x} = \vec{c}$ with

$$A = \begin{pmatrix} 1 & -1 & 5 & -4 \\ 2 & 0 & 4 & -2 \\ 3 & 2 & 0 & 3 \end{pmatrix} \quad \text{and} \quad \vec{c} = \begin{pmatrix} 0 \\ 2 \\ 6 \end{pmatrix}.$$

a) Show that $\vec{c} \notin C(A)$.

b) Compute $A^T A$ and $A^T \vec{c}$.

c) Why does the normal equation $A^T A \widehat{x} = A^T \vec{c}$ not have a unique solution?

d) Find the general solution of the normal equation.

e) Find the projection $\vec{p}$ of $\vec{c}$ onto $C(A)$ and the error vector $\vec{e} \in N(A^T)$.

f) Find the projection matrix P onto $C(A)$, and check that $P\vec{c} = \vec{p}$. (Hint: see question 4.3.28 in the book.)

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[1]See the top of page 210.