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3. (a) False. If $A = I$, then $|A + I| = 4$, but $|A| + 1 = 2$.
   (b) True. $|ABC| = |AB||C| = |A||B||C|$.
   (c) False. If $A = I_2$, then $4A = 16$, but $4|A| = 4$.
   (d) False. Let $A = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$, and $B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Then $AB - BA = \begin{pmatrix} 0 & -2 \\ 3 & 0 \end{pmatrix}$, which doesn't have det 0.

8. (a) $1 = \det(I) = \det(Q^TQ) = \det(Q)\det(Q^T) = \det(Q)^2$. So $\det(Q) = \pm 1$.

9. $|A| = 1$, $|B| = 2$, $|C| = 0$.

10. If the entries of every row add up to 0, then $[1, 1, \dots, 1]^T \in N(A)$. So $\dim N(A) > 0$, so the matrix is singular. If the rows of A add up to 1, then the rows of $A - I$ add up to 0, so $\det(A - I) = 0$ by the previous reasoning. This does not mean $\det A = 1$.

14. The first matrix reduces to $U = \begin{pmatrix} 1 & 2 & 3 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 6 \end{pmatrix}$ so has det $1 \cdot 2 \cdot 3 \cdot 6 = 36$. The second reduces to $\begin{pmatrix} 2 & -1 & 0 & 0 \\ 0 & \frac{3}{2} & -1 & 0 \\ 0 & 0 & \frac{4}{3} & -1 \\ 0 & 0 & 0 & \frac{5}{4} \end{pmatrix}$, so has det $2 \cdot \frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{4} = 5$.

28. (a) True. If A is not invertible, $|A| = 0$, so $|AB| = |A||B| = 0$.
    (b) False. It's $\pm$ product of the pivots, since we may need a row swap to get to an upper triangular matrix.
    (c) False. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = I$. Then $|A| - |B| = -3$, but $|A - B| = -6$.
    (d) True. $|AB| = |B||A| = |A||B| = |AB|$.

29. A may not be square, so may not have a determinant. If it is square, it may have determinant 0.

34. To get B, we'd need one row swap (row 1 swapped with row 3). All other changes are row additions, so $|A| = -|B|$.

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12. $C = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{pmatrix}$. $AC^T = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$.

13. (a) $|C_1| = 0$, $|C_2| = -1$, $|C_3| = 0$, $|C_4| = 1$.

(b) By cofactors, $C_n = 0 \cdot C_{n-1} - 1 \cdot 1 \cdot C_{n-2} = -C_{n-2}$. So $C_{10} = -C_8 = C_6 = -C_4 = -1$.

16.

$$F_n = \begin{vmatrix} 1 & -1 & \\ 1 & \multicolumn{2}{c}{F_{n-1}} \end{vmatrix} = F_{n-1} - (-1) \begin{vmatrix} 1 & -1 & \\ & \multicolumn{2}{c}{F_{n-2}} \end{vmatrix} = F_{n-1} + F_{n-2}$$

34. (a) The first two rows are the same, so the rows are lin. dep.

(b) For every choice of entry in the first row, we can pick a non-zero choice for the second row. For the third row, our choice can be non-zero unless the choices from the first two rows were from columns 4 and 5. By the time we're down to the the fifth row, if we haven't picked a zero yet, then our choices for rows 3 and 4 must have been from columns 4 and 5, which means we must now pick a zero. Therefore all the terms are 0.