MATH 218 SECTION 3
MIDTERM EXAMINATION 1

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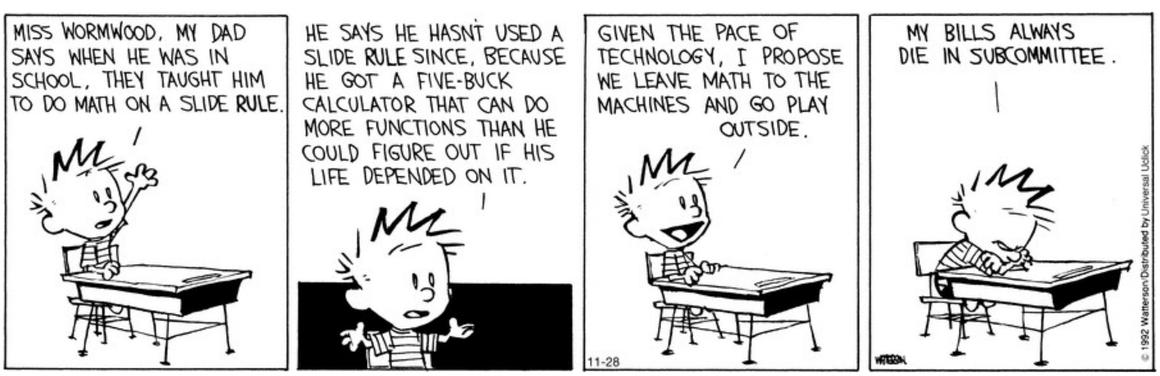
Please **read all instructions** carefully before beginning.

- Do not open this test booklet until you are directed to do so.
- You have 75 minutes to complete this exam.
- If you finish early, go back and check your work.
- This exam is closed book.
- You may use a calculator to do arithmetic, but you should not need one. No other technology is allowed.
- For full credit you must show your work so that your reasoning is clear.
- Do not spend too much time on any one problem. Read them all through first and attack them in an order that allows you to make the most progress.
- Good luck!



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Problem 1.

[15 points]

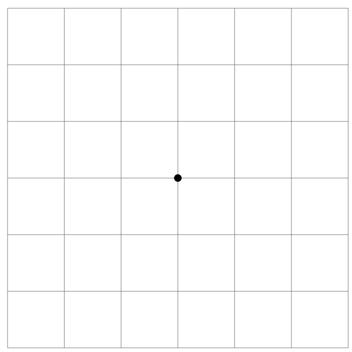
Consider the matrix

$$A = \begin{pmatrix} 1 & -1 & 2 \\ -2 & 2 & -4 \end{pmatrix}.$$

a) Use Gauss–Jordan elimination on A to put A into reduced row echelon form. Circle the free columns.

b) The rank of A is $\boxed{}$.

c) Draw a picture of the column space $C(A)$ below.



d) Find a spanning set for the null space of A.

$$N(A) = S\left\{ \qquad\qquad \right\}$$

e) The null space is a (circle one) $\begin{pmatrix} \text{point} \\ \text{line} \\ \text{plane} \end{pmatrix}$ in (fill in the blank) $\mathbf{R}^{\boxed{}}$.

Problem 2.

[15 points]

Consider the matrix

$$A = \begin{pmatrix} 5 & 4 & 1 \\ -1 & -2 & -1 \\ 0 & -1 & -1 \end{pmatrix}.$$

a) Find the inverse of A.

$$A^{-1} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$

b) Express A^{-1} as a product of elementary matrices.

$$A^{-1} =$$

c) Solve $A\mathbf{x} = \mathbf{b}$, where $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ is an unknown vector.

(Your answer will be a formula in b_1, b_2, b_3.)

$$\mathbf{x} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

Problem 3. [10 points]

Consider the matrix

$$A = \begin{pmatrix} 2 & 3 & 1 \\ -4 & -5 & -3 \\ -2 & -6 & 0 \end{pmatrix}.$$

a) Find a lower-triangular matrix L with ones on the diagonal and an upper-triangular matrix U such that $A = LU$.

$$L = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \qquad U = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$

b) Solve the equation $A\mathbf{x} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$ using the LU decomposition you found above.

$$\mathbf{x} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$$

Problem 4.

[10 points]

a) Briefly explain why a system of linear equations $A\mathbf{x} = \mathbf{0}$ cannot have exactly two solutions.

b) Give an example of a **non-invertible** 3×3 matrix A with **no nonzero entries**.

$$A = \begin{pmatrix} \quad & \quad & \quad \\ & & \\ & & \end{pmatrix}$$

c) Consider the subspace

$$V = S\left\{\begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}1\\-1\end{pmatrix}\right\}.$$

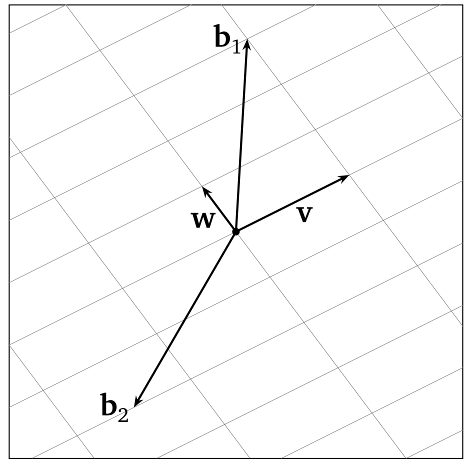
Find two other vectors that span V. (You may not include scalar multiples of the vectors $\binom{1}{1}$ or $\binom{1}{-1}$.)

$$V = S\left\{\begin{pmatrix} \quad \\ \quad \end{pmatrix}, \begin{pmatrix} \quad \\ \quad \end{pmatrix}\right\}.$$

Problem 5.

[10 points]

A certain 2×2 matrix A has columns $\mathbf{v}$ and $\mathbf{w}$, pictured below. Solve the equations $A\mathbf{x}_1 = \mathbf{b}_1$ and $A\mathbf{x}_2 = \mathbf{b}_2$, where $\mathbf{b}_1$ and $\mathbf{b}_2$ are the vectors in the picture.



a) $\mathbf{x}_1 =$

b) $\mathbf{x}_2 =$