MATH 218 SECTION 3
PRACTICE MIDTERM EXAMINATION 1

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Please **read all instructions** carefully before beginning.

- Do not open this test booklet until you are directed to do so.
- You have 75 minutes to complete this exam.
- If you finish early, go back and check your work.
- This exam is closed book.
- You may use a calculator to do arithmetic, but you should not need one. No other technology is allowed.
- For full credit you must show your work so that your reasoning is clear.
- Circle your answers.
- Do not spend too much time on any one problem. Read them all through first and attack them in an order that allows you to make the most progress.
- Good luck!

This is a practice exam. It is meant to be similar in format, length, and difficulty to the real exam. It is **not** meant as a comprehensive list of study problems. I recommend completing the practice exam in 75 minutes, without notes or distractions.

Problem 1. [25 points]

Consider

$$A = \begin{pmatrix} 1 & 2 & 5 & 0 \\ 1 & 2 & 4 & 2 \\ 0 & -1 & 0 & 8 \\ -1 & -3 & -1 & -1 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} 12 \\ 1 \\ -30 \\ 6 \end{pmatrix}.$$

The goal of this question is to solve the equation $A\mathbf{x} = \mathbf{b}$.

a) Carry out Gaussian reduction with maximal partial pivoting to show that the upper triangular component of the $PA = LU$ decomposition is

$$U = \begin{pmatrix} 1 & 2 & 5 & 0 \\ 0 & -1 & 0 & 8 \\ 0 & 0 & 4 & -9 \\ 0 & 0 & 0 & -\frac{1}{4} \end{pmatrix},$$

and find the L and P components. You may want to augment A with the vector $\mathbf{b}$ to make the next part of this question easier.

b) Find $\mathbf{x}$ such that $A\mathbf{x} = \mathbf{b}$.

Problem 2.

[5 points]

Consider the matrix

$$B = \begin{pmatrix} 1 & -1 & 0 \\ -1 & 7 & 1 \\ 2 & 4 & 1 \end{pmatrix}.$$

Is B invertible? If so, find its inverse. If not, explain why.

Problem 3. [15 points]

Consider the matrix

$$D = \begin{pmatrix} 1 & 2 & 3 & 2 & 14 & 9 \\ 0 & 0 & 0 & 2 & 10 & 6 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{pmatrix}.$$

Note that this matrix is in row echelon form.

a) Fill in the blanks:

(1) The column space $C(D)$ is a subspace of $\mathbf{R}^m$, where $m = \boxed{}$.

(2) The null space $N(D)$ is a subspace of $\mathbf{R}^n$, where $n = \boxed{}$.

b) Write down a vector $\mathbf{b}$ such that $D\mathbf{x} = \mathbf{b}$ has no solution. If no such vector exists, explain why not.

c) Compute the reduced row echelon form of D.

d) Find a set of vectors that spans $N(D)$.

Problem 4.

[5 points]

a) Briefly explain why for any two matrices E and F, the column space of EF is contained in the column space of E.

b) Find a matrix F with no zero entries such that the column space of EF is not equal to the column space of E. You do not need to compute the column spaces of either matrix in order to answer this question, but you can use the following matrix if you wish:

$$E = \begin{pmatrix} 1 & 2 & 3 \\ -1 & -2 & 4 \\ 0 & 0 & 7 \end{pmatrix}.$$

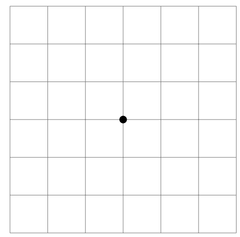
Problem 5.

[10 points]

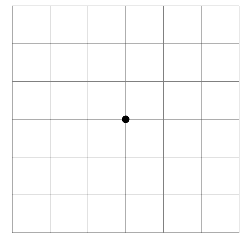
Give examples of 2×2 matrices A, B, C with ranks $0, 1,$ and 2, respectively, and draw pictures of the null space and column space. (Be precise!)

a) Rank 0: $A = \begin{pmatrix} & \\ & \end{pmatrix}$

Nul A

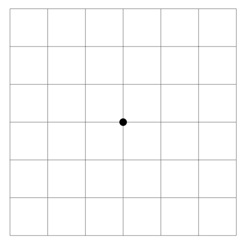


Col A

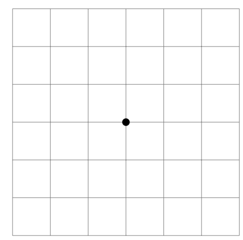


b) Rank 1: $B = \begin{pmatrix} & \\ & \end{pmatrix}$

Nul B

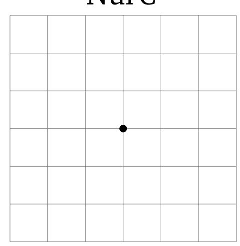


Col B



c) Rank 2: $C = \begin{pmatrix} & \\ & \end{pmatrix}$

Nul C



Col C