MATH 218 SECTION 3
PRACTICE MIDTERM EXAMINATION 2

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Please **read all instructions** carefully before beginning.

- Do not open this test booklet until you are directed to do so.
- You have 75 minutes to complete this exam.
- If you finish early, go back and check your work.
- This exam is closed book.
- You may use a calculator to do arithmetic, but you should not need one. No other technology is allowed.
- For full credit you must show your work so that your reasoning is clear.
- Do not spend too much time on any one problem. Read them all through first and attack them in an order that allows you to make the most progress.
- Good luck!

This is a practice exam. It is meant to be similar in format, length, and difficulty to the real exam. It is **not** meant as a comprehensive list of study problems. I recommend completing the practice exam in 75 minutes, without notes or distractions.

Problem 1. [25 points]

Consider

$$A = \begin{pmatrix} 3 & -3 \\ 3 & 1 \\ 3 & 5 \\ 3 & 1 \end{pmatrix} \qquad \mathbf{v} = \begin{pmatrix} 2 \\ 4 \\ 6 \\ 8 \end{pmatrix}.$$

a) Use Gram–Schmidt to find the QR decomposition of A. You should get

$$Q = \begin{pmatrix} 1/2 & -1/\sqrt{2} \\ 1/2 & 0 \\ 1/2 & 1/\sqrt{2} \\ 1/2 & 0 \end{pmatrix}.$$

b) Find the least squares solution $\widehat{\mathbf{x}}$ of $A\mathbf{x} = \mathbf{v}$, using your QR decomposition above or otherwise.

c) Find the projection $\mathbf{p}$ of $\mathbf{v}$ onto $C(A)$.

Problem 2.

Consider

$$A = \begin{pmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{pmatrix} \qquad \mathbf{b} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}.$$

a) Find the general solution of the system of equations $A\mathbf{x} = \mathbf{b}$.

b) $\dim(N(A)) = \boxed{}$

Problem 3.

a) Suppose that V and W are subspaces of $\mathbf{R}^n$ with $\dim(V) + \dim(W) > n$. Show that there is a nonzero vector contained in both V and W.

b) If P is the projection matrix onto the the row space of a matrix A, explain why $I - P$ projects vectors onto the null space of A.

c) Explain why any set containing the zero vector is linearly dependent.

Problem 4.

Let A be a 4×5 matrix whose null space is spanned by

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 3 \\ 1 \end{pmatrix}.$$

a) Find the projection matrix onto the null space of A.

b) Find the projection matrix onto the row space of A.

Problem 5.

Consider the subspace

$$V = \{\text{all solutions of } 2x + 3y + 4z = 0\}.$$

a) Find a basis for V.

b) Find a basis for $V^{\perp}$.

Problem 6.

A subspace V and a vector $\mathbf{v}$ are drawn below. Draw the projection $\mathbf{p}$ of $\mathbf{v}$ onto V, and draw the projection $\mathbf{p}_{\perp}$ of $\mathbf{v}$ onto $V^{\perp}$. Label your answers!