Linear systems / system of linear equations

Linear equation in the variables $x_1, \dots, x_n$

is an equation:

$$a_1 x_1 + \cdots + a_n x_n = b.$$

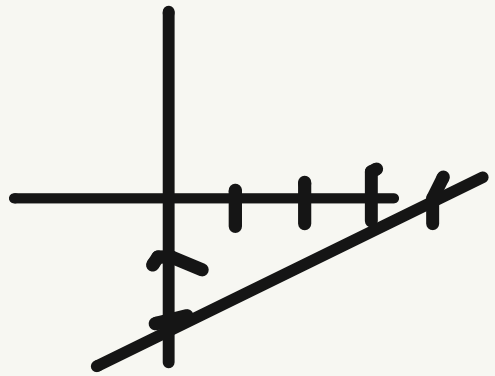
2 variables

Eg $2x + 3y = 5$

$0x + 3y = 5$

Eg $x + y + z = 10$

Eg $x - 2y = 4$

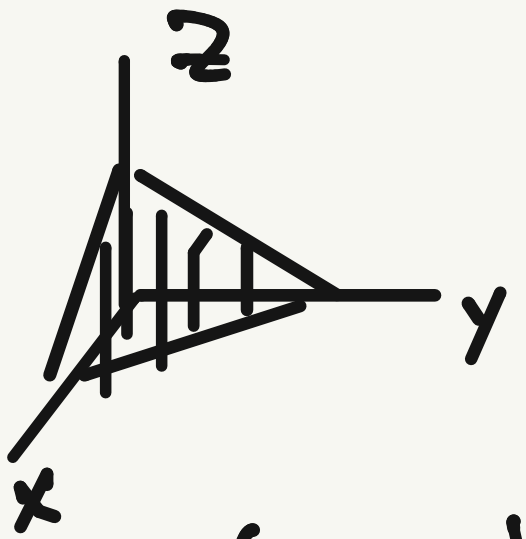


$\mathbb{R}^2$

$(x, y) = (0, -2)$
is one solution

$(x, y) = (6, 1)$ is another

Eg $x+y+z=1$



$\mathbb{R}^3$

$(x,y,z) = \begin{cases} (1,0,0) \\ (1/2, 1/2, 0) \\ (1/3, 1/3, 1/3) \end{cases}$

Eg $x_1 - x_2 + x_3 - x_4 = -1$

$(0, +1, 0, 0)$

Usually

Linear eqn in 2 vars
$\rightsquigarrow$ line

in 3 vars
$\rightsquigarrow$ plane

in n vars
$\rightsquigarrow$ hyperplane

Exception: $0 \cdot x + 0 \cdot y = 0$

Every point in $\mathbb{R}^2$ solves this

Solution $\neq$ line

$0 \cdot x + 0 \cdot y = 1$

No point in $\mathbb{R}^2$ solves

System of linear equs:

$x_1, \dots, x_n$

1^{st} equ $\quad a_{11}x_1 + \cdots + a_{1n}x_n = b_1$

$\vdots \qquad\qquad \vdots$

m^{th} equ $\quad a_{m1}x_1 + \cdots + a_{mn}x_n = b_m$

n variables, m equations

A <u>solution</u> to a linear system is a simultaneous soln

to all eqns.

Eg $x+y=1$
$2x+y=2$

A system of l.n. eqns has either

inconsistent: 1. no solution

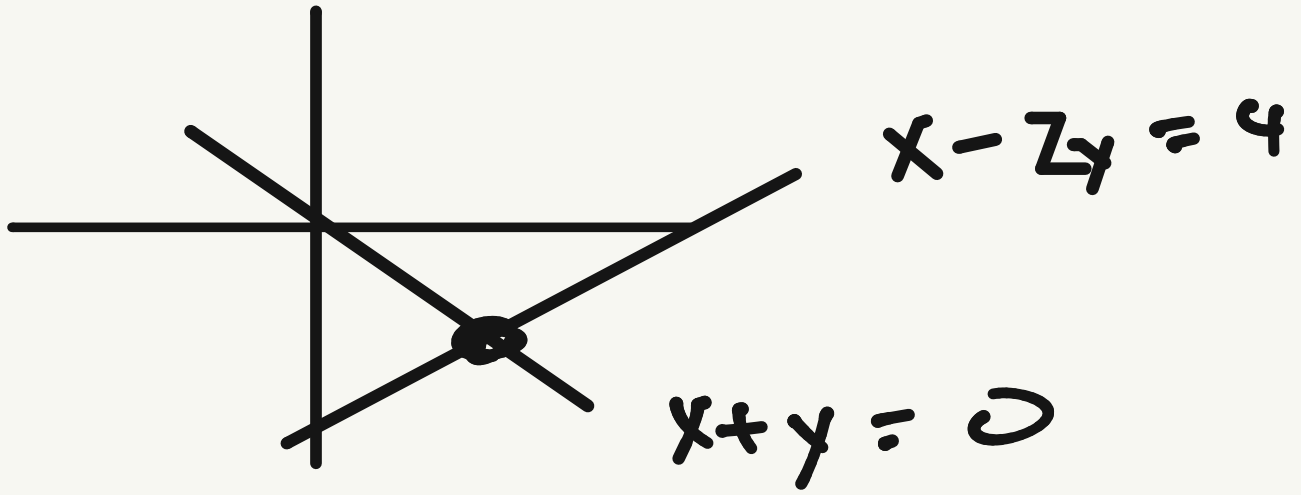
consistent { 2. exactly 1 solution
3. ∞ly many solutions

Eg $0x + 0y = 1$

is an inconsistent

Eg $x - 2y = 4$

$x + y = 0$



$y = -x$

$3x = 4$, $x = 4/3$

$y = -4/3$

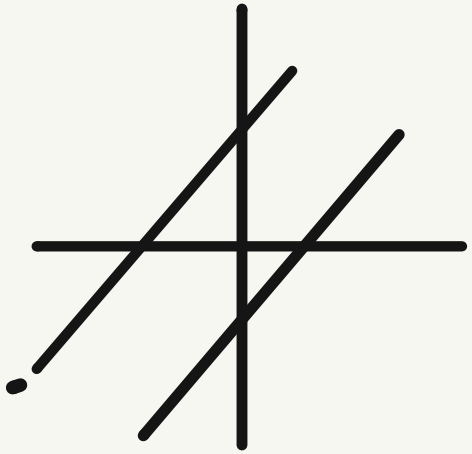
$(x, y) = (4/3, -4/3)$

is the unique solution.

System is consistent.

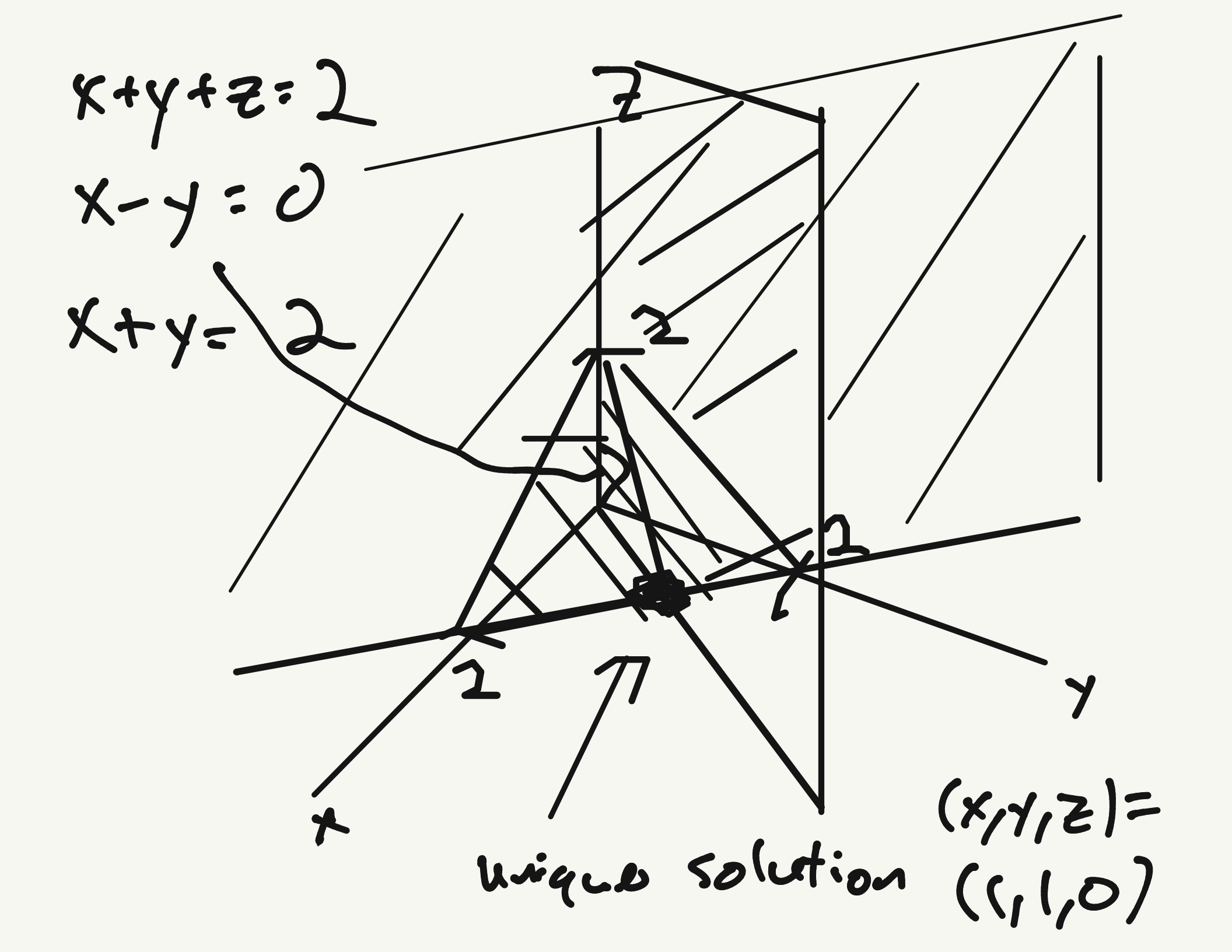
Eg $-x + y = 1$

$-x + y = -1$

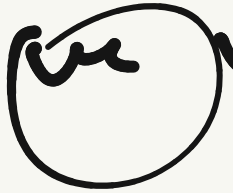


no simultaneous solutions

inconsistent


x+y+z=2
x-y=0
x+y=2
z
2
2
2
2
y
x
unique solution
(x,y,z)=
(1,1,0)

If you have



you can hope a unique solution.

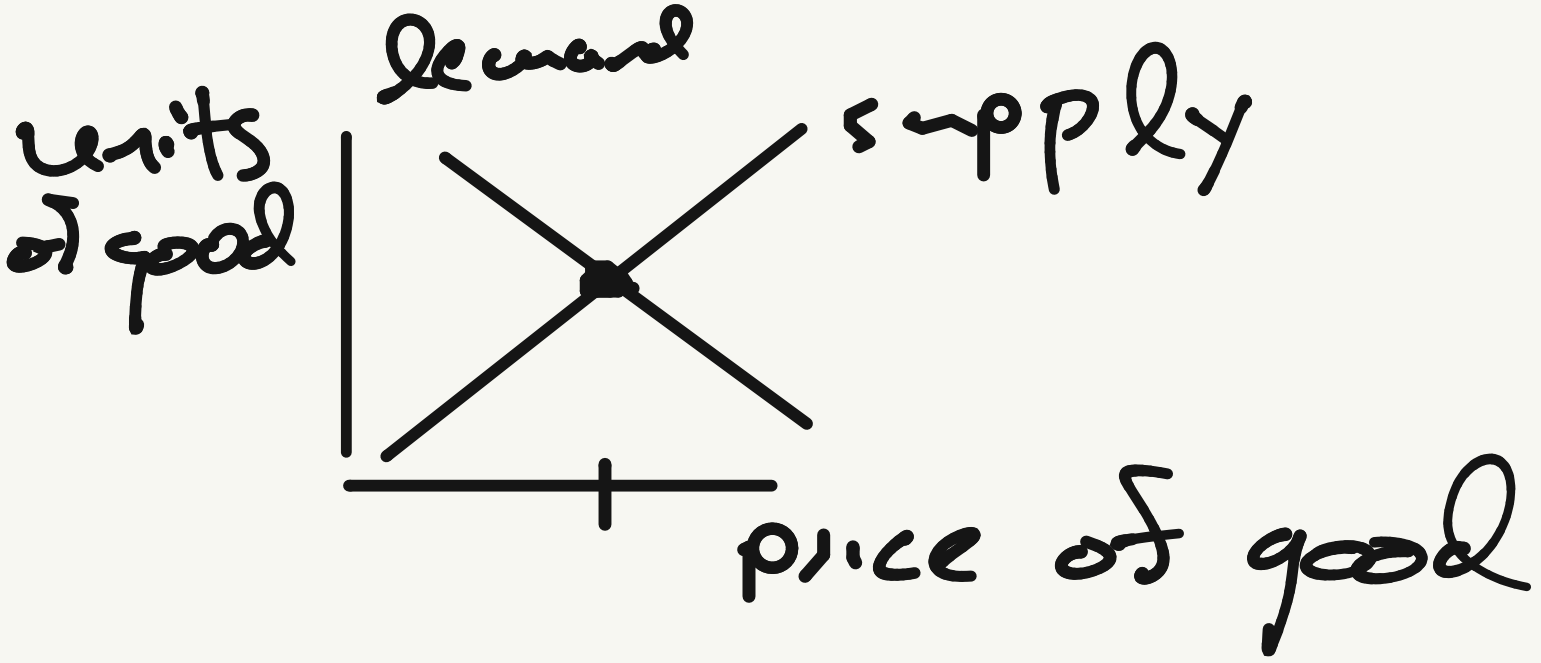
Goal: If you have
m eqns and n vars.

1) does your system actually have a unique solution?

2) if so, describe an algorithm to find unique solution.

Economics, machine learning,
physics, computer vision, ...

Eg Economics



Eg

Phone plan 1: $40 base, $1 per gig

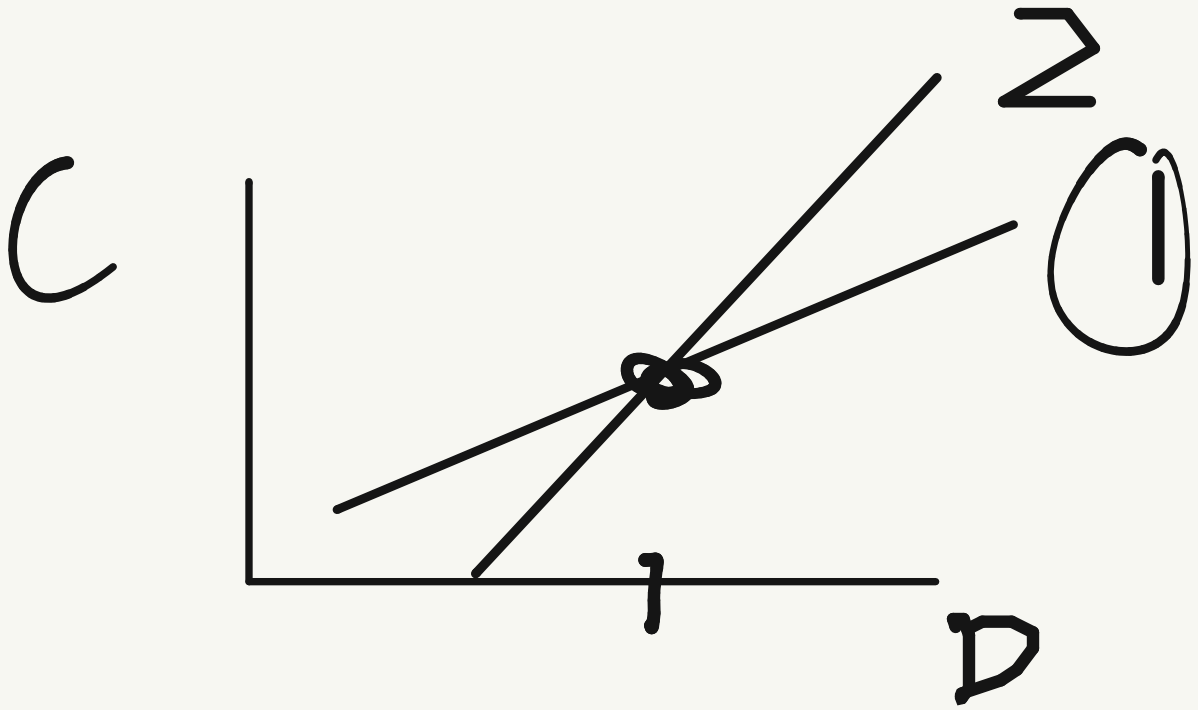
2: $30 base, $2 per gig.

C = cost

D = data used

Plan 1: $C = 40 + 1 \cdot D$

Plan 2: $C = 30 + 2 \cdot D$


C
2
1
1
D