Welcome!

Linear Algebra

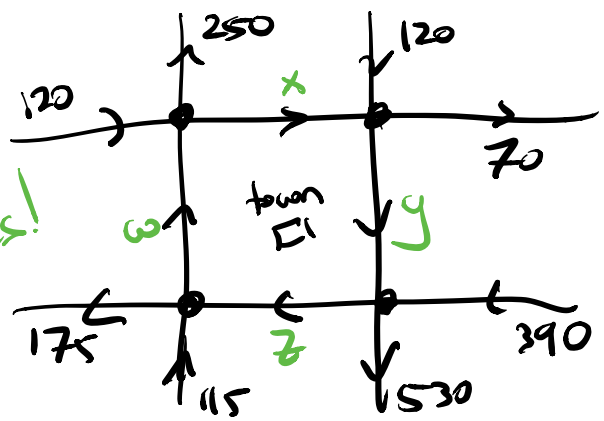
Deals with linear equations

- $y = 3x + 2 \rightsquigarrow -3x + y = 2$
  one eq in 2 variables
- $\begin{cases} x+y+z=1 \\ y-z=-3 \end{cases}$ two eqs in 3 variables

Equations w/ terms that look like
(const) · (variable) or (const)

~~$xy+z=1$~~ ~~$\frac{x}{y}+3=z$~~ ~~$e^x = \cos(y)$~~

Eg:



cars/hr

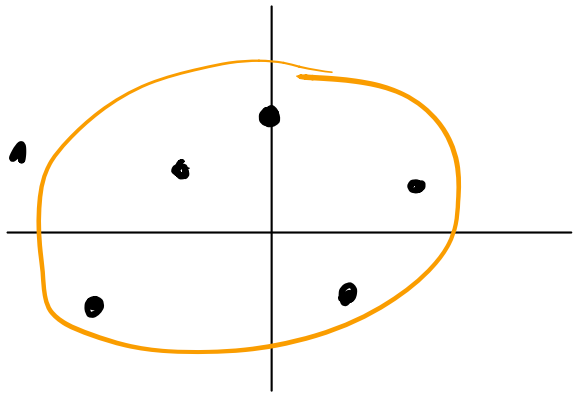
Q: How many cars are on the unlabeled roads?

names!

$$120 + w = 250 + x \rightsquigarrow -x + w = 130$$
$$x + 120 = 70 + y \rightsquigarrow x - y = -50$$
$$y + 390 = 530 + z \rightsquigarrow y - z = 140$$
$$z + 115 = 175 + w \rightsquigarrow z - w = 60$$

4 equations in 4 unknowns!

Eg: An asteroid has been observed at coords $(0,2)$, $(2,1)$, $(1,-1)$, $(-1,-2)$, $(-3,1)$, $(-1,1)$



Q: What is the orbit? Will it crash into Earth?

Orbit is an ellipse

Ellipse: $x^2 + By^2 + Cxy + Dx + Ey + F = 0$

$(0,2)$: $0 + 4B + 0 + 0 + 2E + F = 0$

$(2,1)$: $4 + B + 2C + 2D + E + F = 0$

$(1,-1)$: $1 + B - C + D - E + F = 0$

$\vdots$

$\rightsquigarrow$ 6 eqns in 5 variables B, C, D, E, F

NB: no exact solution!

Q: Best approximate?

Eg: In a population of rabbits:

(1) Half survive their first year

(2) Half of those survive the 2nd year

(3) Max life span is 3 years

(4) Produce 0, 6, 8 offspring in years 1, 2, 3, resp.

Q: How many rabbits will there be after 100 years?

x_{2020} = # 1st year rabbits in 2020

y_{2020} = # 2nd year rabbits in 2020

z_{2020} = # 3rd year rabbits in 2020

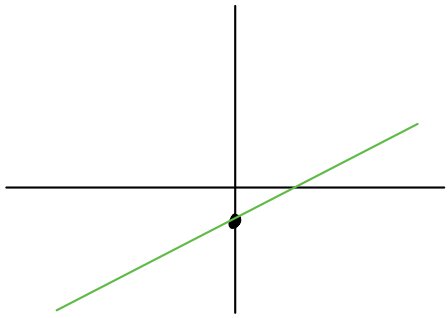
$$\begin{cases} y_{2021} = \frac{1}{2} x_{2020} \\ z_{2021} = \frac{1}{2} y_{2020} \\ w_{2021} = 6y_{2020} + 8z_{2020} \end{cases}$$

difference equation

Geometry of Solutions

- One eqn in 2 vars:

  $x - 2y = 1 \rightsquigarrow y = \frac{1}{2}x - \frac{1}{2}$



- One eqn in 3 vars:

  $x + y + z = 1 \rightsquigarrow z = 1 - x - y$

  plane in xyz-space

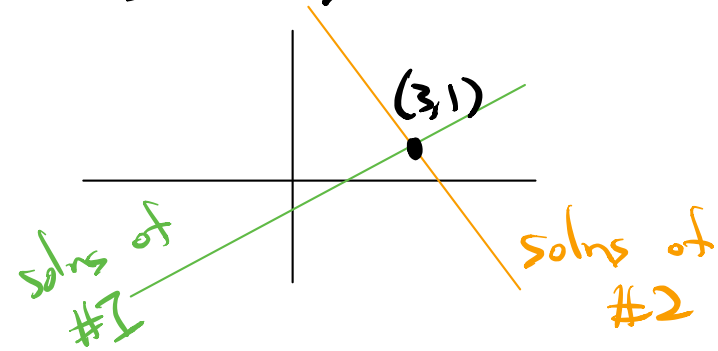
- One eqn in 4 vars:

  $x + y + z + w = 1 \rightsquigarrow$ "3-plane in xyzw-space"

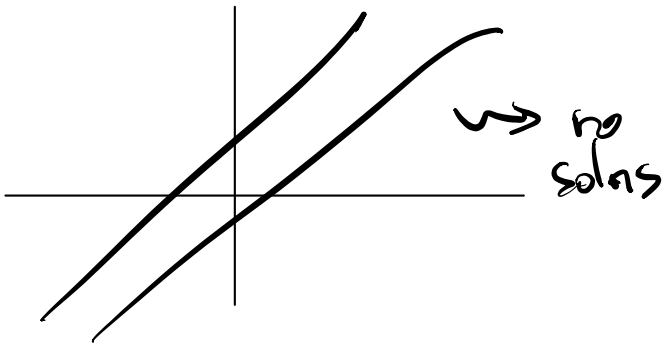
More equations?

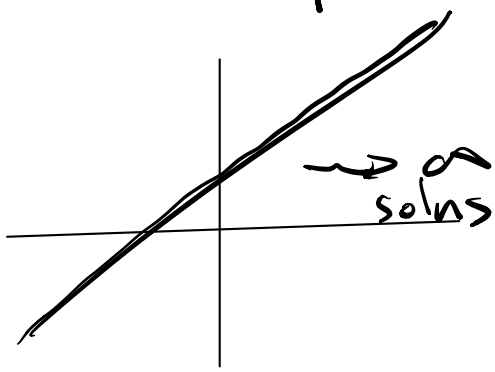
- 2 eqns in 2 vars

  $\left.\begin{aligned} x - 2y &= 1 \\ 3x + 2y &= 11 \end{aligned}\right\}$ simultaneous solutions?



intersect 2 lines: one point





- 2 eqns in 3 vars:

$$\left.\begin{aligned} x+y+z&=1 \\ x\ \ -z&=0 \end{aligned}\right\}$$ intersection of 2 planes in xyz-space: line

- 3 eqns in 3 vars:

$$\left.\begin{aligned} x+y+z&=1 \\ x-z&=0 \\ y&=0 \end{aligned}\right\} \hookrightarrow \begin{aligned} x&=\tfrac{1}{2} \\ y&=0 \\ z&=\tfrac{1}{2} \end{aligned}$$

one point = intersection of 3 planes

Other possibilities?

Administrative

- Course site & Sakai
  → Please turn cameras on!!
- Piazza: tab in Sakai
- Office hours!

- Scan your homework!
  → see Syllabus
- Textbook: • Strang
  • ILA
- Recorded lectures
  → Joe & Jesse.
- Groups & quizzes
- Exams