

Homework #1

due Monday, January 10, at 11:59pm

1. Consider the vectors

$$u = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \quad v = \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix} \quad w = \begin{pmatrix} 8 \\ -6 \\ -2 \end{pmatrix}.$$

- Compute $u + v + w$ and $u + 2v - w$.
 - Find numbers x and y such that $w = xu + yv$.
 - Explain why every linear combination of u, v, w is also a linear combination of u and v only.
 - The sum of the coordinates of any linear combination of u, v, w is equal to _____?
 - Find a vector in \mathbf{R}^3 that is *not* a linear combination of u, v, w .
2. Find two *different* triples (x, y, z) such that

$$x \begin{pmatrix} 1 \\ 2 \end{pmatrix} + y \begin{pmatrix} 1 \\ -2 \end{pmatrix} + z \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}.$$

How many such triples are there?

3. Decide if each statement is true or false, and explain why.
- The vector $\frac{1}{2}v$ is a linear combination of v and w .
 - $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.
 - If v, w are two vectors in \mathbf{R}^2 , then any other vector b in \mathbf{R}^2 is a linear combination of v and w .
4. Suppose that v and w are *unit vectors*: that is, $v \cdot v = 1$ and $w \cdot w = 1$. Compute the following dot products (your answers will be actual numbers):
- $v \cdot (-v)$
 - $(v + w) \cdot (v - w)$
 - $(v + 2w) \cdot (v - 2w)$.
5. Two vectors v and w are *orthogonal* if $v \cdot w = 0$, and they are *parallel* if one is a scalar multiple of the other. A *unit vector* is a vector v with $v \cdot v = 1$.

Decide if each statement is true or false, and explain why.

- If $u = (1, 1, 1)$ is orthogonal to v and to w , then v is parallel to w .
- If u is orthogonal to $v + w$ and to $v - w$, then u is orthogonal to v and w .
- If u and v are orthogonal unit vectors then $(u - v) \cdot (u - v) = 2$.

d) If $u \cdot u + v \cdot v = (u + v) \cdot (u + v)$, then u and v are orthogonal.

6. Find nonzero vectors v and w that are orthogonal to $(1, 1, 1)$ and to each other.
7. Compute the following matrix-vector products using *both* the by-row and by-column methods. If the product is not defined, explain why.

$$\begin{pmatrix} 2 \\ 5 \end{pmatrix} \begin{pmatrix} 1 \\ -3 \\ -1 \end{pmatrix} \quad \begin{pmatrix} 1 & -2 \\ 0 & -1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} \quad \begin{pmatrix} 7 & 2 & 4 \\ 3 & -3 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$
$$\begin{pmatrix} 7 & 4 \\ -2 & 2 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \end{pmatrix} \quad (2 \ 6 \ -1) \begin{pmatrix} 5 \\ -1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 5 \\ -1 \\ 0 \end{pmatrix} (2 \ 6 \ -1)$$

8. Suppose that $u = (x, y, z)$ and $v = (a, b, c)$ are vectors satisfying $2u + 3v = 0$. Find a nonzero vector w in \mathbf{R}^2 such that

$$\begin{pmatrix} x & a \\ y & b \\ z & c \end{pmatrix} w = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

9. Consider the matrices

$$A = \begin{pmatrix} 2 & 1 & -1 \\ 4 & -4 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 5 & 3 & 2 \\ 1 & -1 & 2 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$$
$$D = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} \quad E = (-3 \ 5).$$

Compute the following expressions. If the result is not defined, explain why.

$$\begin{array}{llll} \text{a) } -3A & \text{b) } B - 3A & \text{c) } AC & \text{d) } B^2 \\ \text{e) } A + 2B & \text{f) } C - E & \text{g) } EB & \text{h) } D^2 \end{array}$$

10. Compute the product

$$\begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 2 & 1 & -1 \\ 4 & -1 & 2 \end{pmatrix}$$

in three ways:

- a) Using the column form and the “by columns” method on each column.
b) Using the column form and the “by rows” method on each column.
c) Using the outer product form.

11. Consider the matrices

$$A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 1 \\ -1 & h \end{pmatrix}.$$

What value(s) of h , if any, will make $AB = BA$?

12. Consider the matrices

$$A = \begin{pmatrix} 1 & -3 \\ 2 & 5 \end{pmatrix} \quad B = \begin{pmatrix} -4 & -8 \\ 5 & 8 \end{pmatrix} \quad C = \begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix}.$$

Verify that $AC = BC$ and yet $A \neq B$.

13. For the following matrices A and B , compute $AB, A^T, B^T, B^T A^T$, and $(AB)^T$. Which of these matrices are equal and why? Why can't you compute $A^T B^T$?

$$A = \begin{pmatrix} 1 & 2 \\ -2 & -1 \end{pmatrix} \quad B = \begin{pmatrix} 2 & 1 & -1 \\ 4 & -1 & 2 \end{pmatrix}.$$