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Two nonlinear compactness theorems in $L^p(0, T; B)$

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ABSTRACT

We establish two nonlinear compactness theorems in $L^p(0, T; B)$ with hypothesis on time translations, which are nonlinear counterparts of two results by Simon (1987) [1]. The first theorem sharpens a result by Maitre (2003) [10] and is important in the study of doubly nonlinear elliptic–parabolic equations. Based on this theorem, we then obtain a time translation counterpart of a result by Dubinskiĭ (1965) [5], which is supposed to be useful in the study of some nonlinear kinetic equations (e.g. the FENE-type bead–spring chains model).

1. Introduction

The famous Aubin–Lions–Simon lemma discusses the compactness in $L^p(0, T; B)$ ($1 \le p \le \infty$), which is widely used in the study of nonlinear evolution partial differential equations.

In 1987, Simon [1] generalized the compactness results of Aubin [2] (1963) and Lions [3] (1969) by removing some unnecessary restrictions on spaces such as $1 < p < \infty$ and reflexivity. In [1], for $X \hookrightarrow\hookrightarrow B \hookrightarrow Y$, Simon systematically investigated two types of compactness results in $U \subset L^p(0, T; B)$ with hypothesis on time translations

$$\|\tau_h u - u\|_{L^p(0,T-h;B)} \to 0 \quad \text{as } h \to 0, \text{ uniformly in } u \in U, \ \text{(see [1, Theorem 3])} \tag{1}$$

$$\|\tau_h u - u\|_{L^p(0,T-h;Y)} \to 0 \quad \text{as } h \to 0, \text{ uniformly in } u \in U, \ \text{(see [1, Theorem 5])} \tag{2}$$

and hypothesis on time derivatives

$$\{\partial_t u\}_{u\in U} \text{ is bounded in } L^r(0, T; B), \quad \text{(see [1, Corollary 1])} \tag{3}$$

$$\{\partial_t u\}_{u\in U} \text{ is bounded in } L^r(0, T; Y), \quad \text{(see [1, Corollary 4])} \tag{4}$$

respectively, where $r = 1$ if $1 \le p < \infty$ and $r > 1$ if $p = \infty$. Here and afterwards, unless otherwise specified, X, B, Y are Banach spaces; $(\hookrightarrow\hookrightarrow) \hookrightarrow$ denotes (compact) continuous embedding; $(\tau_h u)(t) := u(t + h)$ for $h > 0$, where u is a vector-valued function. Moreover, it seems that Simon is also the first author to discuss the compactness theorem with hypothesis on time translations (see [4]). We refer to the compactness result with hypothesis (1) or (2) as the time translation compactness theorem and the result with hypothesis (3) or (4) as the time derivative compactness theorem, respectively.

In 1965, Dubinskiĭ [5] established a compactness theorem with a hypothesis on time derivatives in $L^p(0, T; B)$, where B is a normed linear space. In Dubinskiĭ's compactness theorem, X is replaced by a seminormed set M (*not a linear subspace*) of B, where $M \hookrightarrow\hookrightarrow B \hookrightarrow Y$ (Y is a normed linear space). So in contrast to Corollary 4 of Simon [1], Dubinskiĭ's theorem

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can be regarded as a kind of nonlinear compactness theorem. In addition, Dubinskiĭ [5] used this theorem to study the weak solutions of degenerate quasi-linear parabolic equations.

Recently, Barrett and Süli [6] have corrected some minor errors of Dubinskiĭ's nonlinear time derivative compactness theorem in [5], filled in some missing details (some of them are nontrivial) of its proof and obtained a similar result in $L^p(0, T; B)$, where X is replaced by a seminormed nonnegative cone M_+ of B and $M_+ \hookrightarrow\hookrightarrow B \hookrightarrow Y$. Moreover, Barrett and Süli [7,8] applied the Dubinskiĭ's theorem in [6] to study the weak solutions of FENE-type and Hookean-type bead–spring chains model.

In the study of doubly nonlinear elliptic–parabolic equations, the compactness theorems in [1] cannot be directly applied. In 2003, motivated by a nonlinear compactness argument of Alt and Luckhaus [9] and Theorem 1 of Simon [1], Maitre [10] obtained a nonlinear compactness theorem in $L^p(0, T; B)$ with hypothesis on time translations. In Maitre's theorem, X is replaced by a *nonlinear subset* $\mathcal{B}(X)$ of B, where $\mathcal{B} : X \to B$ is a nonlinear compact operator. There is an unnatural technical assumption, saying the boundedness in $L^r_{\mathrm{loc}}(0, T; B)$ with $r > 1$ in [10]. However, in a counterpart compactness result, Theorem 3 of Simon [1], r is taken to be 1. Barrett and Süli [6] revealed that Maitre's nonlinear time translation compactness theorem implies Dubinskiĭ's nonlinear time derivative compactness theorem.

In this paper, we will establish two nonlinear compactness theorems with hypothesis on time translations instead of hypothesis on time derivatives. Indeed, the time translation (nonlinear) compactness theorem has two advantages in contrast with the time derivative one. First, the former implies the latter. Second, when using a semi-discretization in time scheme to construct approximate solutions of the nonlinear evolution PDE, one can apply a very simple time translation compactness theorem given by Dreher and Jüngel [11] directly to avoid using linear interpolation functions (also known as Rothe functions, see [12,13]) and hence make the discussion more clean. More precisely, if $u_k\,(1 \le k \le N)$ are the approximate solutions with step length $\tau = \frac{T}{N}$ for an evolution PDE and denote that $u_\tau(t, \cdot) = u_k,\ t \in ((k-1)\tau, k\tau],\ k = 1, 2, \ldots, N$, then Dreher and Jüngel [11, Theorem 1] show a simple time translation assumption

$$\|\tau_\tau u_\tau - u_\tau\|_{L^r(0,T-\tau;Y)} \le C\tau, \tag{5}$$

where $r = 1$ if $1 \le p < \infty$ and $r > 1$ if $p = \infty$. This is a simplified and more applicable version of (2).

In Section 2, we sharpen Maitre's result (see [10, Theorem 2.1]) by replacing $r > 1$ by $r = 1$ and obtain a nonlinear counterpart of Simon's result (see [1, Theorem 3]), which is important in the study of doubly nonlinear elliptic–parabolic equations. Based on Maitre's method, more delicate and concise analysis of multi-integrals is used in our proof.

In Section 3, using Theorem 1 in Section 2, we present a time translation counterpart of Dubinskiĭ's nonlinear time derivative result (see [6, Theorem 1]), which is also a nonlinear counterpart of Simon's time translation result (see [1, Theorem 5]). This nonlinear time translation compactness theorem is supposed to be useful in the study of some nonlinear kinetic equations (e.g., the FENE-type bead–spring chains model).

2. The first nonlinear time translation compactness theorem

The operator $\mathcal{B} : X \to B$ is called a (nonlinear) compact operator, if it maps bounded subsets of X to relatively compact subsets of B. Let $L^1_{\mathrm{loc}}(0, T; X)$ be the set of functions f such that for any $0 < t_1 < t_2 < T$, $f \in L^1(t_1, t_2; X)$, equipped with the semi-norms $\|f\|_{L^1(t_1, t_2;X)}$. A subset V of $L^1_{\mathrm{loc}}(0, T; X)$ is called bounded, if for any $0 < t_1 < t_2 < T$, V is bounded in $L^1(t_1, t_2; X)$.

Theorem 1. *Let X, B be Banach spaces, $1 \le p \le \infty$ and $\mathcal{B} : X \to B$ be a (nonlinear) compact operator. Assume that V is a bounded subset of $L^1_{\mathrm{loc}}(0, T; X)$ such that $U = \mathcal{B}(V) \subset L^p(0, T; B)$ and*

$$U \text{ is bounded in } L^1_{\mathrm{loc}}(0, T; B), \tag{6}$$

$$\|\tau_h u - u\|_{L^p(0,T-h;B)} \to 0 \quad \text{as } h \to 0, \text{ uniformly for } u \in U. \tag{7}$$

Then U is relatively compact in $L^p(0, T; B)$ (and in $C([0, T]; B)$ if $p = \infty$).

Remark 2. In Maitre's Theorem (see [10, p. 1726, Theorem 2.1]), they assume that U is bounded in $L^r_{\mathrm{loc}}(0, T; B)$ with $r > 1$. This condition is the key of his proof (see [10, pp. 1727–1728]). Here we replace this unnatural assumption $r > 1$ by $r = 1$. This makes Theorem 1 a nonlinear counterpart of Simon's time translation compactness theorem (see [1, p. 80, Theorem 3]).

Proof. In view of Theorem 1 on page 71 of Simon [1], we only need to show that for any $0 < a < b < T$,

$$P = \left\{ \int_a^b u(t)dt : u \in U \right\} \text{ is relatively compact in } B. \tag{8}$$

Note that a and b will be fixed all the time in the following.

Step 1. For any $v \in V$ and $M > 0$, we define a subset of $[a, b]$,

$$E^M_v = \{t \in [a, b] : \|v(t)\|_X > M\}.$$

Since V is bounded in $L^1(a, b; X)$, there exists a constant $C > 0$ such that for any $v \in V$, $\|v\|_{L^1(a,b;X)} \leq C$. Then E_v^M is a Lebesgue measurable subset of $[a, b]$ with bounds on its measure

$$m(E_v^M) \leq \frac{C}{M}, \quad \forall v \in V. \tag{9}$$

Define

$$v^M(t) = \begin{cases} v(t), & \text{if } t \notin E_v^M \\ 0, & \text{if } t \in E_v^M. \end{cases} \tag{10}$$

Therefore

$$\forall M > 0,\ \forall v \in V,\ \forall t \in [a, b], \quad \|v^M(t)\|_X \leq M. \tag{11}$$

Moreover, we have from (10) that for any $u = \mathcal{B}(v)$ and $t \in [a, b]$,

$$u^M(t) := \mathcal{B}(v^M(t)) = \chi_{[a,b]\setminus E_v^M}(t)u(t) + \chi_{E_v^M}(t)\mathcal{B}(0). \tag{12}$$

Step 2. We shall prove that

$$\begin{aligned} &\forall \varepsilon > 0,\ \exists M \in \mathbb{N},\ \text{such that } \forall u = \mathcal{B}(v) \in U,\ \exists s_u^M \in (0, h),\ \text{such that} \\ &\left\| \int_a^b u(t)dt - \sum_{k=1}^N u^M(t_{k-1} + s_u^M)h \right\|_B < \varepsilon, \\ &\text{where } N = \left[\sqrt{M}\right],\ h = \frac{b-a}{N} \text{ and } t_k = a + kh,\ k = 0, 1, \ldots, N. \end{aligned} \tag{13}$$

For this purpose, we shall first claim that

$$I := \frac{1}{h}\int_0^h \left\| \int_a^b u(t)dt - \sum_{k=1}^N u^M(t_{k-1} + s)h \right\|_B ds \leq 2 \sup_{\sigma \in [0,h]} \|u(\cdot + \sigma) - u\|_{L^1(a,b-\sigma;B)} + \frac{C}{\sqrt{M}}. \tag{14}$$

Indeed,

$$\begin{aligned} I &= \frac{1}{h}\int_0^h \left\| \sum_{k=1}^N \int_{t_{k-1}}^{t_k} [u(t) - u^M(t_{k-1} + s)]dt \right\|_B ds \leq \frac{1}{h}\sum_{k=1}^N \int_0^h \int_{t_{k-1}}^{t_k} \|u(t) - u^M(t_{k-1} + s)\|_B dtds \\ &= \frac{1}{h}\sum_{k=1}^N \int_{t_{k-1}}^{t_k} \int_{t_{k-1}}^{t_k} \|u(t) - u^M(\tau)\|_B dtd\tau. \end{aligned}$$

Hence we obtain from (12) that

$$\begin{aligned} I \leq\ & \frac{1}{h}\sum_{k=1}^N \int_{t_{k-1}}^{t_k} \int_{t_{k-1}}^{t_k} \|u(t) - u(\tau)\|_B dtd\tau + \frac{1}{h}\sum_{k=1}^N \int_{t_{k-1}}^{t_k} \left[\chi_{E_v^M}(\tau) \int_{t_{k-1}}^{t_k} \|u(t) - \mathcal{B}(0)\|_B dt \right] d\tau \\ :=\ & I_1 + I_2. \end{aligned}$$

First setting $\sigma = t - \tau$ in the inner integral and then using Fubini's theorem, we deduce that

$$\begin{aligned} I_1 &= \frac{1}{h}\sum_{k=1}^N \int_{t_{k-1}}^{t_k} \int_{t_{k-1}-\tau}^{t_k-\tau} \|u(\tau + \sigma) - u(\tau)\|_B d\sigma d\tau \\ &= \frac{1}{h}\sum_{k=1}^N \int_{-h}^0 \int_{t_{k-1}-\sigma}^{t_k} \|u(\tau + \sigma) - u(\tau)\|_B d\tau d\sigma + \frac{1}{h}\sum_{k=1}^N \int_0^h \int_{t_{k-1}}^{t_k-\sigma} \|u(\tau + \sigma) - u(\tau)\|_B d\tau d\sigma \\ &\leq \frac{1}{h}\int_{-h}^0 \int_{a-\sigma}^b \|u(\tau + \sigma) - u(\tau)\|_B d\tau d\sigma + \frac{1}{h}\int_0^h \int_a^{b-\sigma} \|u(\tau + \sigma) - u(\tau)\|_B d\tau d\sigma. \end{aligned}$$

By setting $-\sigma = \sigma$ and then setting $\tau - \sigma = t$, we have

$$\frac{1}{h}\int_{-h}^0 \int_{a-\sigma}^b \|u(\tau + \sigma) - u(\tau)\|_B d\tau d\sigma = \frac{1}{h}\int_0^h \int_a^{b-\sigma} \|u(t) - u(t + \sigma)\|_B dtd\sigma.$$

Therefore

$$I_1 \le \frac{2}{h}\int_0^h \int_a^{b-\sigma} \|u(\tau+\sigma)-u(\tau)\|_B d\tau d\sigma \le 2 \sup_{\sigma\in[0,\,h]} \|u(\cdot+\sigma)-u\|_{L^1(a,\,b-\sigma;B)}.$$

Since U is bounded in $L^1(a,b;B)$, there exists a constant $C>0$ such that for any $u\in U$, $\|u\|_{L^1(a,\,b;B)} \le C$. Hence, it follows from (9) and the definition of N and h that

$$I_2 \le \frac{1}{h}\int_a^b \chi_{E_v^M}(\tau)d\tau \cdot \|u(t)-\mathcal{B}(0)\|_{L^1(a,\,b;B)} \le \frac{N}{b-a} m(E_v^M)\Big(\|u\|_{L^1(a,\,b;B)} + \|\mathcal{B}(0)\|_B(b-a)\Big) \le \frac{C}{\sqrt{M}}.$$

Therefore, we establish (14) and hence in light of (7) that

$$\frac{1}{h}\int_0^h \left\| \int_a^b u(t)dt - \sum_{k=1}^N u^M(t_{k-1}+s)h \right\|_B ds \to 0 \quad \text{as } M\to+\infty,\ \text{uniformly for } u\in U. \tag{15}$$

By noting that

$$\forall M\in\mathbb{N},\ \forall u\in U,\ \exists s_u^M\in(0,\ h) \text{ such that, } \quad \left\| \int_a^b u(t)dt - \sum_{k=1}^N u^M(t_{k-1}+s_u^M)h \right\|_B$$
$$\le \frac{1}{h}\int_0^h \left\| \int_a^b u(t)dt - \sum_{k=1}^N u^M(t_{k-1}+s)h \right\|_B ds,$$

(13) follows.

Step 3. Set

$$P^M = \left\{ \sum_{k=1}^N u^M(t_{k-1}+s_u^M)h : u^M = \mathcal{B}(v^M),\ v\in V \right\}.$$

$B(0,\varepsilon)$ denotes an open ball of B centered at the origin with the radius ε. Eq. (13) reads $P\subset B(0,\varepsilon)+P^M$. Since for fixed M and for any $v\in V$, (11) implies that $v^M(t_{k-1}+s_u^M)$ is bounded in X, one has from the compactness of $\mathcal{B}$ that P^M is relatively compact in B.

Therefore, for every $\varepsilon>0$, there exists P^M, which is a relatively compact ε-net in B for P. This yields (8) and finishes the proof of Theorem 1. □

3. The second nonlinear time translation compactness theorem

In the following three paragraphs, we recall some definitions in [6]. Let $M_+\subset B$. If $\forall\, u\in M_+, \forall c\in[0,\infty),\ cu\in M_+$, then M_+ is called a nonnegative cone in B. If in addition, there exists a function $[u]_{M_+}: M_+\to\mathbb{R}$ such that

$$[u]_{M_+}\ge 0; \qquad [u]_{M_+}=0 \quad \text{if and only if} \quad u=0;$$
$$\forall c\in[0,\infty), \quad [cu]_{M_+} = c[u]_{M_+},$$

then M_+ is called a seminormed nonnegative cone in B.

A seminormed nonnegative cone M_+ in B is said to be continuously embedded in B, if there exists a constant C, such that for any $u\in M_+$, $\|u\|_B\le C[u]_{M_+}$. The embedding is called compact, if for any infinite bounded set of elements in M_+, there exists a subsequence which converges in B.

Denote by $L^p(0,T;M_+)(1\le p<\infty)$ the set of all vector-valued functions $u:[0,T]\to M_+$ such that $\int_0^T [u]_{M_+}^p dt < \infty$. Then $L^p(0,T;M_+)$ is a seminormed nonnegative cone in $L^p(0,T;B)$ with $[u]_{L^p(0,T;M_+)} = \left(\int_0^T [u]_{M_+}^p dt\right)^{1/p}$. Likewise, define the seminormed nonnegative cone $L^\infty(0,T;M_+)$ and $C([0,T];M_+)$ with $\|u\|_{L^\infty(0,T;M_+)} = \text{ess.sup}_{t\in[0,T]}[u]_{M_+}$ and $\|u\|_{C([0,T];M_+)} = \max_{t\in[0,T]}[u]_{M_+}$, respectively.

Theorem 3. *Let B, Y be Banach spaces, $1\le p\le\infty$ and M_+ be a seminormed nonnegative cone in B. Assume $M_+\hookrightarrow\hookrightarrow B\hookrightarrow Y$ and*

$$U \text{ is a bounded subset of } L^p(0,T;M_+), \tag{16}$$
$$\|\tau_h u-u\|_{L^p(0,T-h;Y)}\to 0 \quad \text{as } h\to 0,\ \text{uniformly for } u\in U. \tag{17}$$

Then U is relatively compact in $L^p(0,T;B)$ (and in $C([0,T];B)$ if $p=\infty$).

Remark 4. This theorem is a time translation counterpart of Dubinskiĭ's nonlinear time derivative result (see [6, Theorem 1]) and also a nonlinear counterpart of Simon's time translation compactness theorem (see [1, p. 84, Theorem 5]).

We divide the proof of Theorem 3 into two lemmas.

Lemma 5 (*See [5, p. 612, Lemma 1] or [6, Lemma 1]*). *Let B, Y be Banach spaces and M_+ be a seminormed nonnegative cone in B. Assume $M_+ \hookrightarrow\hookrightarrow B \hookrightarrow Y$. Then for any $\varepsilon > 0$, there exists a constant $C_\varepsilon > 0$ such that*

$$\forall u, v \in M_+, \quad \|u - v\|_B \le \varepsilon([u]_{M_+} + [v]_{M_+}) + C_\varepsilon \|u - v\|_Y.$$

Lemma 6. *Let Y be a Banach space, $1 \le p \le \infty$ and M_+ be a seminormed nonnegative cone in Y. Assume $M_+ \hookrightarrow\hookrightarrow Y$ and*

$$U \text{ is a bounded subset of } L^p(0, T; M_+), \tag{18}$$

$$\|\tau_h u - u\|_{L^p(0,T-h;Y)} \to 0 \quad \text{as } h \to 0, \text{ uniformly for } u \in U. \tag{19}$$

Then U is relatively compact in $L^p(0, T; Y)$ (and in $C([0, T]; Y)$ if $p = \infty$).

Proof. We might as well assume $1 \le p < \infty$. The proof for $p = \infty$ is similar and hence will be omitted.

Define

$$\mathcal{B}(v) = \begin{cases} \dfrac{\|v\|_Y}{[v]_{M_+}} v, & \text{if } v \in M_+ \setminus \{0\} \\ 0, & \text{if } v \in \Big(Y \setminus M_+\Big) \cup \{0\}. \end{cases}$$

Let $\{v_n\}_{n\in\mathbb{N}}$ be a bounded sequence in Y, then $\mathcal{B}(v_n) \in M_+$ and $[\mathcal{B}(v_n)]_{M_+} \le \|v_n\|_Y$. Hence $\{\mathcal{B}(v_n)\}_{n\in\mathbb{N}}$ is a bounded sequence in M_+. In light of $M_+ \hookrightarrow\hookrightarrow Y$, we have that $\{\mathcal{B}(v_n)\}_{n\in\mathbb{N}}$ is relatively compact in Y. Therefore $\mathcal{B} : Y \to Y$ is a nonlinear compact operator.

For each $w \in U$, define

$$v(t) = \begin{cases} \dfrac{[w(t)]_{M_+}}{\|w(t)\|_Y} w(t), & \text{on } \Big\{t \in [0, T] : w(t) \ne 0\Big\} := [w(t) \ne 0] \\ 0, & \text{on } \Big\{t \in [0, T] : w(t) = 0\Big\} := [w(t) = 0] \end{cases} \tag{20}$$

and $V = \Big\{v \text{ defined as (20)} : w \in U\Big\}$. Then for any $v \in V$,

$$\|v\|_{L^p(0,T;Y)} = \left(\int_{[w(t)\ne 0]} [w(t)]_{M_+}^p \, dt\right)^{\frac{1}{p}} \le \|w\|_{L^p(0,T;M_+)}.$$

Thus we have from (18) that V is a bounded subset in $L^p(0, T; Y)$ and hence in $L^1(0, T; Y)$.

Set

$$\widetilde{U} := \mathcal{B}(V) = \Big\{u(t) = \mathcal{B}(v(t)), t \in [0, T] : v \in V\Big\}.$$

Then for any $u = \mathcal{B}(v) \in \widetilde{U}$, we have from the continuous embedding $M_+ \hookrightarrow Y$ that

$$\|u\|_{L^p(0,T;Y)} = \left(\int_{[v(t)\ne 0]} \left\| \frac{\|v(t)\|_Y}{[v(t)]_{M_+}} v(t) \right\|_Y^p dt\right)^{\frac{1}{p}} \le C\|v\|_{L^p(0,T;Y)}.$$

Therefore $\widetilde{U}$ is also a bounded subset in $L^p(0, T; Y)$ and hence in $L^1(0, T; Y)$. It follows from Theorem 1 that $\widetilde{U}$ is relatively compact in $L^p(0, T; Y)$.

Next, we prove $\widetilde{U} = U$. Indeed, for any $w \in U$, define a $v \in V$ as (20), then

$$u(t) = \mathcal{B}(v(t)) = \begin{cases} \dfrac{\|v(t)\|_Y}{[v(t)]_{M_+}} v(t), & \text{on } [v(t) \ne 0] = [w(t) \ne 0] \\ 0, & \text{on } [v(t) = 0] = [w(t) = 0]. \end{cases}$$

$$= \begin{cases} w(t), & \text{on } [w(t) \ne 0] \\ 0, & \text{on } [w(t) = 0] \end{cases} = w(t) \quad \text{on } [0, T].$$

This ends the proof of Lemma 6. □

Proof of Theorem 3. Based on Lemmas 5 and 6, the proof of Theorem 3 is similar as Lemma 9 in [1]. □

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