

Course syllabus for Math 690-10

Stable homotopy theory

Fall 2025

Course description: Certain problems such as classifying manifolds up to cobordism are stable in the sense that they are solved in categories where it is possible to desuspend. Other problems, such as classifying algebraic vector bundles on a space, require analogous unstable information. Even if your goal is to solve unstable problems, it is frequently useful to solve the stable version of the problem first and then destabilize. This course will introduce stable homotopy theory and the category of spectra. Spectra have duals, and Poincaré duality becomes the statement that manifolds are twisted self-dual in the category of spectra. We will introduce generalized cohomology theories, and study the examples of K-theory, cobordism and, time permitting, Morava E and K-theories, which are higher height analogues of K-theory.

Some references: *Stable Homotopy and Generalized Homology* by J.F. Adams. Available at <https://www.sas.rochester.edu/mth/sites/doug-ravenel/otherpapers/Adams-SHGH-latex2.pdf>

Time and place: WF 10:05-11:20 am. Gross Hall 324. We will not have class Friday October 17, or Friday October 24, 2025. Instead we will have optional make-up classes the week of December 1-5.

Instructor contact information and office hours: Kirsten Graham Wickelgren, e-mail: kirsten.wickelgren@duke.edu,

office: Physics and Math: room 025

office hours: subject to change based on weekly constraints. Wednesdays 9-10am, Friday 11:20am-12:20pm or by appointment.

Prerequisites: A semester of graduate level algebraic topology is prerequisite.

Topics:

- Spectra. Smash product.
- Spanier-Whitehead duality. Poincaré duality.
- Generalized (co)homology theories. Vector bundles and K-theory, (co)bordism. Atiyah-Hirzebruch spectral sequence. (We will introduce spectral sequences too, so you are not required to know how to use them.)
- Adams Spectral sequence.
- J-homomorphism.

Assignments: All undergraduates must take an individual oral exam before midsemester grades and shortly before the end of class. All registered students are expected to show up to class. Math PhD students are required to hand in 5 exercises by the end of the term, one of which must be handed in by the midsemester grade deadline. Math PhD students who choose to additionally do an oral exam or a presentation will receive a 3D-printed math art object!

Grading: Grades will be based on the requirements listed in the assignments section. For undergraduates, the midsemester oral exam will be weighted as 40% and the final oral exam will be weighted as 60%. A student's grade may be lowered in response to excessive class absence.

Collegiality statement: Please read the collegiality statement on the course website.

Questionnaire: Please fill out the Questionnaire. It is available on the course website, but I prefer having paper copies.

Duke University's Community Standard is available at: <https://studentaffairs.duke.edu/conduct/about-us/duke-community-standard>