Course syllabus for Math 8803
Stable homotopy theory

Spring 2015

Course description: Certain problems such as classifying manifolds up to cobordism are stable in the sense that they are solved in categories where it is possible to desuspend. Other problems, such as classifying algebraic vector bundles on a space, require analogous unstable information. Even if your goal is to solve unstable problems, it is frequently useful to solve the stable version of the problem first and then destabilize. This course will introduce stable homotopy theory and the category of spectra. Spectra have duals, and Poincaré duality becomes the statement that manifolds are twisted self-dual in the category of spectra. We will introduce generalized cohomology theories, and study the examples of K-theory, and Morava E and K-theories, which are higher height analogues of K-theory.

Some references: Stable Homotopy and Generalized Homology by J.F. Adams. *Model Categories of Diagram Spectra* by M.A. Mandell, J.P. May, S. Schwede, and B. Shipley. There will be course notes available on the webpage. Please see the notes for further references.

Time and place: MWF 1:00-2:00 pm. Skiles 270. Webpage through link at `http://www.math.gatech.edu/~kwickelgren3`. Some information will be on T-square too.

Instructor contact information and office hours: Kirsten Graham Wickelgren, e-mail: kwickelgren3@math.gatech.edu, office: Skiles 227. office hours: Monday at 12pm, Wednesday at 2pm when there is no student seminar and at 3pm when there is a student seminar, or by appointment.

Prerequisites: A semester of graduate level algebraic topology is prerequisite.

Topics:

- Spectra. Smash product.

- Spanier-Whitehead duality. Poincaré duality.
- Generalized (co)homology theories. Vector bundles and K-theory, (co)bordism. Atiyah-Hirzebruch spectral sequence. (We will introduce spectral sequences too, so you are not required to know how to use them.)
- Adams Spectral sequence.
- J-homomorphism.

Assignments: There will be optional exercises available with the course notes. Registered students are expected to show up to class, and give a presentation in the Wednesday Geometry-Topology Student seminar on a topic related to the course. There is a list of suggestions below. Please discuss your presentation topic with me. If you would rather not give a presentation, come talk to me, and we will decide on an alternative e.g. the optional assignments could substitute. There will be no exams.

Grading: Grades will be based on the requirements listed in the assignments section. As one might expect, failure to complete the presentation (or the alternative discussed above) will result in a failing grade. Similarly, students who stop attending class should also expect a low or failing grade.

The Georgia Institute of Technology honor code is available at: `http://www.catalog.gatech.edu/rules/18b.php`

Some suggested topics for presentations:

- James construction.
- Hilton-Milnor theorem.
- Clifford algebras and vector fields on spheres.
- Milnor's theorem on the dual Steenrod algebra.
- L-theory and the Hirzebruch signature formula.
- Quillen's theorem that $\pi_* MU$ is the universal formal group law L, called the *Lazard ring*.
- Orientations and Thom spectra. (One example: complex orientations are classified by maps from MU. A different topic along these lines is the Atiyah-Bott-Shapiro KO-orientation of Spin bundles. Presenting *Units of Ring Spectra and Thom Spectra* by Ando, Blumberg, Gepner, Hopkins and Rezk is a third possibility.)

- Adam's operations and the étale homotopy type of BGL_n. Application: algebraic K-theory of finite fields.
- Nilpotence Theorem.
- Kervaire invariant 1 problem.