Minicourse
L-theory, bilinear forms, and surgery

Spring 2021

Course description: L-theory is a cohomology theory formed from non-degenerate symmetric bilinear or quadratic forms modulo cobordism. So forms related by a surgery give the same element of L-theory. Examples of L-groups include the classical Witt groups of fields and rings. Poincaré duality on compact oriented manifolds determines elements of L-theory of Z, and manifolds related by a geometric surgery give bilinear forms related by an algebraic surgery. The Hirzebruch signature formula can be viewed as a comparison between two orientations on the smash product of L with ordinary cohomology with Q coefficients. This minicourse will introduce L-theory and discuss these topics. It is inspired by recent work of B. Calmès, E. Dotto, Y. Harpaz, F. Hebestreit, M. Land, K. Moi, D. Nardin, T. Nikolaus and W. Steimle (`https://arxiv.org/abs/2009.07223`, `https://arxiv.org/abs/2009.07224`, `https://arxiv.org/abs/2009.07225`), building on work of J. Lurie.

Topics:

- K-theory to L-theory
- Grothendieck–Witt groups and Witt groups, examples
- Witt groups and classical bilinear forms to Poincare objects (first look)
- Derived categories and stable homotopy theories
- Quadratic functors
- Poincare objects and surgery
- geometric surgery and Poincare duality on manifolds
- L-groups, L-theory spaces and spectra
- Hirzebruch signature theorem

- Comments on recent work and future directions

References:

- M. Land Lecture notes *New Perspectives on K and L-theory* https://www.uni-muenster.de/MathematicsMuenster/events/2020/K-theory-2020.shtml
- J. Lurie Course notes *Algebraic L-theory and Surgery* https://www.math.ias.edu/~lurie/287x.html
- F. Binda and M. Morrow *Aspects of Motivic Cohomology* https://www.ias.edu/sites/default/files/Morrow\%20lectures\%201\%2B2.pdf
- B. Calmès, E. Dotto, Y. Harpaz, F. Hebestreit, M. Land, K. Moi, D. Nardin, T. Nikolaus and W. Steimle *Hermitian K-theory for Stable Infinity Categories I II and III*(https://arxiv.org/abs/2009.07223, https://arxiv.org/abs/2009.07224, https://arxiv.org/abs/2009.07225)
- W. Course notes *Math 8803 Stable homotopy theory* https://services.math.duke.edu/~kgw/8803_Stable/

Time and place: Tu - Th 1:45PM - 3:00PM, 08/24/2021 to 09/21/2021, Physics 205

Website: https://services.math.duke.edu/~kgw/minicourse21-Ltheory/

Instructor contact information: Kirsten Graham Wickelgren,

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office: Physics and Math: room 025

office hours: Tuesday, Thursday 3:00-3:30, or by appointment

Prerequisites: algebra, homotopy theory.

Collegiality statement: Please read the collegiality statement on the course webpage.

Duke University's Community Standard is available at: https://studentaffairs.duke.edu/conduct/about-us/duke-community-standard