ERRATUM TO "THE SIMPLICIAL EHP SEQUENCE IN $\mathbb{A}^1$–ALGEBRAIC TOPOLOGY"

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1. Mistake in proof of Lemma 3.3

As pointed out by Aravind Asok, the proof of Lemma 3.3 is not quite correct as written. The statement of the lemma is, fortunately, correct.

Lemma 3.3. *Let X be an object of $\mathbf{sPre}(\mathbf{Sm}_k)$, let s be a point of $\mathbf{Sh}_{\mathrm{Nis}}(\mathbf{Sm}_k)$ and let P be a set of primes. Then $s^*L_PX \simeq L_Ps^*X$.*

Proof. We first claim that s^*L_PX is P–local. Since it is fibrant, it suffices to show that if ρ^k_n is an element of T_P, then the induced map

$$(\rho^k_n)_* : \mathrm{SMap}(S^k_\tau, s^*L_PX) \to \mathrm{SMap}(S^k_\tau, s^*L_PX)$$

is a weak equivalence. If $\{U_i\}$ is a system of neighbourhoods for s^* then there is a succession of natural isomorphisms

$$\begin{aligned}\mathrm{SMap}(S^k_\tau, s^*L_PX) &\cong \mathrm{SMap}(S^k_\tau, \operatorname*{colim}_U (L_PX)(U)) \\ &\cong \operatorname*{colim}_U \mathrm{SMap}(S^k_\tau, (L_PX)(U)) \qquad \textbf{since } S^k_\tau \textbf{ is compact,} \\ &\cong \operatorname*{colim}_U \mathrm{SMap}(S^k_\tau \times U, L_PX)\end{aligned}$$

and ρ^k_n induces a weak equivalence on the spaces $\mathrm{SMap}(S^k_\tau \times U, L_PX)$ since L_PX is P-local.
The functor s^* preserves trivial cofibrations, and therefore the map $s^*X \to s^*L_PX$ is a trivial cofibration the target of which is fibrant in the P-local model structure on **sSet**. Therefore s^*L_PX is weakly equivalent in the ordinary model structure on **sSet** to any other P-fibrant-replacement for s^*X, notably to L_Ps^*X, which is what was claimed. □

The mistaken part is highlighted in red. While S^k, the simplicial k-sphere defined by $\Delta[k]/\partial\Delta[k]$, is a finite simplicial set, and therefore compact, the object S^k_τ is a fibrant replacement and consequently not finite. The fix is to observe that the homotopy type of the mapping object $\mathrm{SMap}(S^k_\tau, \mathrm{colim}_U(L_PX)(U)$ is invariant under replacing the source by an equivalent object, in this case the finite S^k. The corrected proof reads:

Corrected proof of Lemma 3.3.

Lemma 3.3. *Let X be an object of $\mathbf{sPre}(\mathbf{Sm}_k)$, let s be a point of $\mathbf{Sh}_{\mathrm{Nis}}(\mathbf{Sm}_k)$ and let P be a set of primes. Then $s^*L_PX \simeq L_Ps^*X$.*

Proof. We first claim that s^*L_PX is P–local. Since it is fibrant, it suffices to show that if ρ^k_n is an element of T_P, then the induced map

$$(\rho^k_n)_* : \mathrm{SMap}(S^k_\tau, s^*L_PX) \to \mathrm{SMap}(S^k_\tau, s^*L_PX)$$

is a weak equivalence. If $\{U_i\}$ is a system of neighbourhoods for s^* then there is a succession of natural weak equivalences

$$\begin{aligned} \mathrm{SMap}(S^k_\tau, s^* L_P X) &\cong \mathrm{SMap}(S^k_\tau, \operatorname*{colim}_U (L_P X)(U)) \\ &\simeq \mathrm{SMap}(S^k, \operatorname*{colim}_U (L_P X)(U)) \\ &\cong \operatorname*{colim}_U \mathrm{SMap}(S^k, (L_P X)(U)) \qquad \text{since } S^k \text{ is compact,} \\ &\simeq \operatorname*{colim}_U \mathrm{SMap}(S^k_\tau, (L_P X)(U)) \\ &\cong \operatorname*{colim}_U \mathrm{SMap}(S^k_\tau \times U, L_P X) \end{aligned}$$

and ρ^k_n induces a weak equivalence on the spaces $\mathrm{SMap}(S^k_\tau \times U, L_P X)$ since $L_P X$ is P-local.

The functor s^* preserves trivial cofibrations, and therefore the map $s^* X \to s^* L_P X$ is a trivial cofibration the target of which is fibrant in the P-local model structure on **sSet**. Therefore $s^* L_P X$ is weakly equivalent in the ordinary model structure on **sSet** to any other P-fibrant-replacement for $s^* X$, notably to $L_P s^* X$, which is what was claimed. □

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