Math 601
Groups, Rings, and Fields

Spring 2021

Course description:

- Groups: Elementary concepts (homomorphism, subgroup, coset, normal subgroup, simple group). Solvable groups, commutator subgroup, Sylow theorems, structure of finitely generated Abelian groups. Symmetric, alternating, dihedral, and general linear groups.
- Rings: Commutative rings and ideals (principal, prime, maximal). Chinese remainder theorem. Integral domains, factorization, principal ideal domains, Euclidean domains, polynomial rings, Gauss Lemma,Eisenstein's irreducibility criterion.
- Modules: Elementary concepts: homomorphism, linear independence, exact sequence, finite presentation, torsion. Structure of finitely generated modules over a principal ideal domain.
- Fields: Extensions: finite, algebraic, separable, inseparable, transcendental, splitting field of a polynomial, primitive element theorem, algebraic closure. Finite Galois extensions and the Galois correspondence between subgroups of the Galois group and subextensions. Solvable Galois groups and the problem of expressing the roots of a polynomial in terms of radicals. Finite fields.

Approximate Schedule:

- 8/24: Group homomorphisms, group actions
- 8/26: Orbits, stabilizers
- 8/31: Cosets, Lagrange, Orbit-Stabilizer
- 9/2: Normal subgroups, quotient groups

- 9/7: isomorphism theorems
- 9/9 : class equation
- 9/14: Simplicity A_n
- 9/16: Sylow theorems
- 9/21: Burnside's lemma, semi-direct products
- 9/23: Classification of finitely generated abelian groups
- 9/28: Composition series
- 9/30: Midterm 1: group theory
- 10/5: no class Fall break
- 10/7: Rings
- 10/12: Localization, UFD
- 10/14: Factorization, polynomial rings
- 10/19: Hilbert basis
- 10/21: Modules, rational canonical form
- 10/26: Modules over a PID
- 10/28: Jordan canonical form
- 11/2: Midterm 2: rings and module theory
- 11/4: Fields
- 11/9: Splitting fields, classification of finite fields
- 11/11: algebraic closures
- 11/16: Separability
- 11/18: Galois theory
- 11/23 Galois theory II

Text: Abstract Algebra, 3rd Edition by David S. Dummit, Richard M. Foote John Wiley and Sons

Time and place: Tu - Th 3:30PM - 4:45 PM, 08/24/2021 to , Physics 205

Website: on Sakai

Assignments: There will be be two problem sets on group theory, two on ring theory, one on module theory, and two on field theory. There will be at least one week to do each problem set, although usually there will be more time. You are encouraged to work together on the homework, but solutions are to be written up independently. You are also free to consult outside references to complete the homework, but solutions are to be written up in your own words. An unfortunate consequence of allowing the use of outside resources is that some students use them heavily to do the homework and then have difficulty with exams. It is not practical to forbid the use of outside resources, and it can also be helpful, but please restrict their use to that which will increase your understanding of abstract algebra. Any sources you use or collaborators you consult should be credited in writing on your work, of course. Assignments will be distributed and submitted using Gradescope. A selection of problems will be graded.

There will be two in-class midterms. The first will be on group theory. The second will be on ring theory, and module theory. Although some group theory may appear on the second midterm, it will not be the emphasis. The first midterm is tentatively scheduled for September 30, and the second is tentatively scheduled for November 2. In the case of absences from midterms for Duke official business, religious holidays, illness, and the like, the final exam will count for the midterm and the final in the final grade.

The final will cover all the material in the course and is on Thursday, December 9, 2:00-5:00pm in Physics 205.

Instructor contact information: Kirsten Graham Wickelgren,

e-mail: kirsten.wickelgren@duke.edu,

office: Physics and Math: room 025

office hours: Tuesday, Thursday 3:00-3:30 (you can find me in room 205 at 3pm, or I will walk down to my office), Monday 1-2pm, or by appointment

Prerequisites: Students who take Math 601 should have taken a semester long course in abstract algebra (e.g. Math 501 at Duke) and should be familiar with the following topics: linear algebra, equivalence relations, equivalence classes, the rudiments of group theory including subgroups, homomorphisms, group actions; the bare rudiments of ring theory including ring homomorphisms and ideals.

Collegiality statement: Please read the collegiality statement on the course website.

Questionnaire: Please fill out the Questionnaire available on the course website.

Duke University's Community Standard is available at: `https://studentaffairs.duke.edu/conduct/about-us/duke-community-standard`