

Course syllabus for Math 605

Algebraic Number Theory

Spring 2026

Course description: The fundamental theorem of arithmetic says that any integer can be expressed uniquely as a product of primes (up to reordering and units). The class group of a number field measures the failure of this theorem for finite field extensions of \mathbb{Q} . Gauss conjectured that there are infinitely many fields $\mathbb{Q}[\sqrt{d}]$ with unique factorization. This is still unknown. Algebraic number theory studies such questions and this course introduces algebraic number theory.

Some references: References for the material in this course include Neukirch *Algebraic Number Theory*, Lang *Algebraic Number Theory*, Serre *Local Fields*, Silverman *Advanced Topics in the Arithmetic of Elliptic Curves*, and Brian Conrad's notes for Math 676.

Time and place: TuTh 11:45AM - 1:00AM Physics 205

Instructor contact information and office hours: Kirsten Graham Wickelgren, e-mail: kirsten.wickelgren@duke.edu,

office: Physics and Math: room 025

office hours: subject to change based on weekly constraints. Wednesdays 12:30-1:30pm, Tuesday 1-2 or by appointment.

Prerequisites: Abstract algebra as in Duke Mathematics 502 or 601.

Topics:

- Binary quadratic forms;
- Dedekind domains, global fields, integral closure;

- Class groups, orders, fractional ideals;
- CM elliptic curves
- Unit groups
- valuations and local fields;
- ramification;
- zeta functions;
- L-functions and class number formulas.

Assignments: All undergraduates must take an individual oral exam before midsemester grades and shortly before the end of class. All registered students are expected to show up to class. Course requirements for enrolled graduate students will be decided as a class, based on the individual interests of the students.

Grading: Grades will be based on the requirements listed in the assignments section. For undergraduates, the midsemester oral exam will be weighted as 40% and the final oral exam will be weighted as 60%. A student's grade may be lowered in response to excessive class absence.

Collegiality statement: Please read the collegiality statement on the course website.

Questionnaire: Please fill out the Questionnaire. It is available on the course website, but I prefer having paper copies.

Duke University's Community Standard is available at: <https://studentaffairs.duke.edu/conduct/about-us/duke-community-standard>