LEGEND FOR ATLAS OF TWO-COMPONENT LEGENDRIAN LINKS

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The table represents a first approximation to a classification of Legendrian links in all prime two-component link types of arc index up to 9.

- Links are **unoriented** but **ordered**, with the first component in red (and thicker, for black and white printouts) and the second in blue (and thinner).
- The grid diagrams shown are non-destabilizable, and conjectured (at least for many of the knot types given) to comprise all possible non-destabilizable Legendrian representatives of the appropriate topological type, modulo switching the components where possible (see below). Each grid diagram is accompanied by the ordered pair (tb_1, tb_2) of Thurston–Bennequin numbers for its components. Different grid diagrams are conjectured but **not guaranteed** to represent distinct Legendrian links.
- The "TB Polyhedron" column depicts the set of possible ordered pairs (tb_1, tb_2) for a Legendrian link in that topological type. More precisely, the possible ordered pairs are the lattice points on the dashed lines, along with all lattice points below and to the left of the dashed lines. (The latter points can be achieved via stabilization of one or both components.) See also next item.
- For links where the TB Polyhedron is all black, the depicted region has been proven to be precisely the set of possible ordered pairs (tb_1, tb_2). TB Polyhedra with red corners are conjectural. In all cases, the region determined by the dashed lines is provably an upper bound; that is, no ordered pair above or to the right of the dashed lines can be attained. This is proven using the Khovanov bound for the Thurston–Bennequin numbers of the individual components, along with their union. In some cases one needs a further bound, which can be provided by applying the Khovanov bound to a three-component link given by doubling one of the two given components. The black dots indicate ordered pairs that are known to be attained (by the figures in the table), while the red dots are conjectured.
- The "Topological Switch?" column marks whether the two components of the topological link can be switched with each other via ambient isotopy. Dashes in this column indicate that the components cannot be switched for the trivial reason that they represent distinct knot types.

- For topological links where the components can be switched, only Legendrian representatives with $tb_1 \leq tb_2$ are shown. One can switch the Legendrian components to get other Legendrian representatives that may or may not be different (see next item).
- The "Legendrian Switch?" column marks whether the two components of the Legendrian link can be switched via Legendrian isotopy. Dashes in this column indicate that the components cannot be switched for one of the following trivial reasons: either they cannot be switched topologically, or they have distinct Thurston–Bennequin numbers. X's in this column are mainly marked with question marks, indicating that we believe that the Legendrian components cannot be switched (with a superscript indicating the maximal size of grid diagram checked by the program). For L7a6, one can use linearized contact homology to prove that the Legendrian components cannot be switched.
- One can show in various nontrivial cases that diagrams shown in the atlas are not Legendrian isotopic. In some cases (e.g., the two L6a3 links with $(tb_1, tb_2) = (-3, -3)$), links can be distinguished by rotation number. Less obviously, linearized contact homology distinguishes the two Legendrian L6a1 links, as well as the two L7a4 links. It would be interesting to know to what extent various Legendrian invariants can distinguish the links in the table.