Mathematics 401: Introduction to Abstract Algebra

Fall 2012 | Tu,Th 10:05–11:20 am | Physics building 119

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Course synopsis: The integers and modular arithmetic; polynomials, their roots, and field extensions; groups and symmetry. Applications may include the symmetries of plane tessellations, the impossibility of trisecting an angle with compass and straightedge, the symmetries of platonic solids, modern cryptography, and/or a brief introduction to Galois theory. An important goal of the course is to learn to write clear and correct mathematical proofs. Note: this is the new numbering for the former Math 121.

Course web site: All important information (homework, handouts, etc.) will be posted on Sakai, `http://sakai.duke.edu/`. There is also a rudimentary web site at `http://www.math.duke.edu/~ng/math401/`, but I don't expect to use this.

Textbook: The required text for this course is *Abstract Algebra: A Geometric Approach* by Theodore Shifrin. I strongly recommend that you read the text concurrently with lectures as we go along.

Office hours: Two hours per week TBA, and by appointment (set up in person or by email). Please take full advantage of office hours to resolve any questions you may have about course material or homework. If you want to set up an appointment via email outside of scheduled office hours, please keep in mind that I can't usually answer email immediately; on occasion it may take a day for me to respond.

Prerequisites: Math 221 (linear algebra, old Math 104).

Alternate course: A more advanced version of our course is Math 501 (old Math 200), which can be extended to a yearlong course by adding Math 502 (old Math 201), but you can't take Math 501 if you've already taken Math 401, so please plan carefully.

Assignments: There will be weekly homework sets due in class on Thursdays. I will announce each assignment in class on the previous Thursday, and on Sakai. Please take care to make your homeworks neat and legible, and remember to staple multi-page submissions. You are allowed and encouraged to work with fellow students on the homework; however, each student must write up their problem sets on their own. If you've collaborated with someone, please mention this fact (and their name) on your homework, for full disclosure. *No late homework will be accepted* except in university-sanctioned cases.

Quizzes: There will be weekly quizzes on the days that homework assignments are due. These will occupy the last ten minutes of class, and will consist of one problem (or portion of a problem) taken from the problem set due that day. Your lowest quiz score will be dropped.

Exams: There will be two midterm exams and a final exam. The final is **Sunday December 16, 2:00–5:00 pm**. The midterms will be held in class and are *tentatively* scheduled for **Thursday September 27** and **Thursday November 1**.

Grading: Your grade will be based on a weighted average of your grades as follows:

- Homework 5%
- Quizzes 10%
- Each midterm 25%
- Final exam 35%.

Topics to be covered: Here is a tentative list of topics, time permitting and subject to change.

- Logic, sets, functions, equivalence relations (sections A.1, A.2, A.3)
- Induction, integers, prime numbers, Euclidean algorithm, Fundamental Theorem of Arithmetic, modular arithmetic (sections 1.1, 1.2, 1.3)
- Rings, integral domains, fields, $\mathbb{Z}_m$, $\mathbb{C}$ (sections 1.4 and 2.3)
- Polynomial rings, division algorithm, remainder theorem, root-factor theorem, Euclidean algorithm for polynomials, unique factorization (section 3.1)
- Roots of polynomials, Fundamental Theorem of Algebra, adjoining elements, Rational Root Theorem, Gauss's lemma (sections 3.2 and 3.3)
- Ring homomorphisms, ideals, isomorphisms, homomorphism theorem, splitting fields (sections 4.1 and 4.2)
- Gaussian integers, primes of the form $a^2 + b^2$, primes of the form $4k + 1$ (section 4.3)
- Groups, symmetry groups, group homomorphisms and isomorphisms (sections 6.1 and 6.2)
- Cosets, Lagrange's theorem, classification of small finite groups, normal subgroups, quotient groups, fundamental homomorphism theorem (section 6.3)
- Group actions, orbits, stabilizers, symmetry groups of regular polyhedra (sections 7.1 and 7.2).